导热规律为 $Q \propto \Delta(\frac{1}{T})$ 时卡诺热机的 $(\eta P)_{max}$ *

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〔摘要〕 本文研究了导热规律为 $Q \propto \Delta(\frac{1}{T})$ 时卡诺热机效率与功率并重的工作状态 $(\eta P)_{max}$，导出了该态下的 η_m、P_m，并与同一导热规律下的最大输出功率(P_{max})态进行了比较。

关键词　导热规律　热机　性能优化
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0 引言

有限时间热力学提出以来，不少学者对热机的性能进行了单目标优化的研究，得到了许多有意义的结论。但单目标优化往往不能兼顾热机其它性能函数。文献[1]研究了遵从牛顿传热规律时卡诺热机效率与功率乘积的最大值 $(\eta P)_{max}$，讨论表明，$(\eta P)_{max}$ 是热机的一个重要参数，它表示了对效率与功率并重的最佳工作状态。本文将研究传热规律为 $Q \propto \Delta(\frac{1}{T})$ 时卡诺热机效率与功率并重的最佳工作状态 $(\eta P)_{max}$，求出该态的效率 η_m，功率 P_m，并作有益的讨论。

1 基本优化关系及 P_{max}、$\eta_{m\cdot R}$

本文研究的卡诺热机仍采用内可逆卡诺循环模型。设高、低温热源的温度分别为 T_H、T_L；循环中工质的吸、放热温度为 T_1、T_2；吸、放热时间为 t_1、t_2；工质与高、低温热源间的传热系数分别为 α、β，则循环中工质的吸、放热量为

$$Q_1 = \alpha(\frac{1}{T_1} - \frac{1}{T_H})t_1 \tag{1}$$

$$Q_2 = \beta(\frac{1}{T_L} - \frac{1}{T_2})t_2 \tag{2}$$

从热机输出功率及效率的基本公式 $P = \frac{Q_1 - Q_2}{\tau}$ 和 $\eta = 1 - \frac{Q_2}{Q_1}$ 出发，利用式(1)、(2)求得输出功率与循环效率间的基本优化关系为[2]

$$P = \frac{\alpha\eta[(1-\eta)T_L^{-1} - T_H^{-1}]}{[1 + \sqrt{\frac{\alpha}{\beta}}(1-\eta)]^2} = \frac{\alpha}{T_L}\frac{\eta(1-\eta-\theta)}{[1+\delta(1-\eta)]^2} \tag{3}$$

其中 $\delta = \sqrt{\frac{\alpha}{\beta}}$，$\theta = \frac{T_L}{T_H}$。应用极值条件，从式(3)求得其最大输出功率及其对应的最佳循环效率分别为[2]

$$\eta_{m\cdot R} = \frac{(1+\delta)(1-\theta)}{\delta(1+\theta)+2} \tag{4}$$

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$$P_{max} = \frac{\alpha}{4T_L} \cdot \frac{(1-\theta)^2}{(1+\delta)(1+\delta\theta)} \quad (5)$$

2 热机的 $(\eta P)_{max}$ 及 η_m、P_m

对传热律 $Q \propto \Delta(\frac{1}{T})$ 下效率与功率并重的工作状态的性能进行优化分析时，用式(3)将效率及功率的乘积写为

$$\eta P = \frac{\alpha}{T_L} \cdot \frac{\eta^2(1-\eta-\theta)}{[1+\delta(1-\eta)]^2} \quad (6)$$

应用极值条件 $\partial(\eta P)/\partial\eta = 0$ 可以求得当循环效率为

$$\eta_m = \frac{3(1+\delta)}{2\delta}\left(1-\sqrt{1-\frac{8}{9}\frac{\delta}{1+\delta}(1-\theta)}\right)$$

$$= \frac{3(1+\delta)}{2\delta}\left(1-\sqrt{1-\frac{8}{9}\frac{\delta}{1+\delta}\eta_c}\right) \quad (7)$$

时，功率与效率的乘积取极大值。式中 $\eta_c = 1 - \theta = 1 - \frac{T_L}{T_H}$ 为可逆卡诺热机的效率。ηP 的极大值为

$$(\eta P)_{max} = \frac{\alpha}{T_L} \cdot \frac{\eta_m^2(1-\eta_m-\theta)}{[1+\delta(1-\eta_m)]^2} \quad (8)$$

这时输出功率为

$$P_m = \frac{\alpha}{T_L} \frac{\eta_m(1-\eta_m-\theta)}{[1+\delta(1-\eta_m)]^2} \quad (9)$$

3 比较与讨论

3.1 与最大输出功率态优化性能的比较

先将效率与功率并重的工作状态与最大输出功率态的优化性能进行比较。为简单计，考虑 $\alpha = \beta$，即 $\delta = 1$ 的情况。此时

$$\eta_m = 3\left(1-\sqrt{\frac{5}{9}+\frac{4}{9}\theta}\right) \quad (10)$$

$$(\eta P)_{max} = \frac{\alpha}{T_L} \times \frac{9\left(1-\sqrt{\frac{5}{9}+\frac{4}{9}\theta}\right)^2\left(3\sqrt{\frac{5}{9}+\frac{4}{9}\theta}-2-\theta\right)}{\left(3\sqrt{\frac{5}{9}+\frac{4}{9}\theta}-1\right)^2} \quad (11)$$

$$P_m = \frac{\alpha}{T_L} \times \frac{3\left(1-\sqrt{\frac{5}{9}+\frac{4}{9}\theta}\right)\left(3\sqrt{\frac{5}{9}+\frac{4}{9}\theta}-2-\theta\right)}{\left(3\sqrt{\frac{5}{9}+\frac{4}{9}\theta}-1\right)^2} \quad (12)$$

定义

$$\gamma = \eta_m/\eta_{m \cdot R} \quad (13)$$

$$\kappa = P_m/P_{max} \quad (14)$$

当 $\delta = 1$ 时可得

$$\lim_{\theta\to 0} \gamma = \frac{9\left(1-\sqrt{\frac{5}{9}}\right)}{2} \doteq 1.1458$$

$$\lim_{\theta\to 1} \gamma = \frac{4}{3} \quad (15)$$

$$\lim_{\theta\to 0} \kappa \doteq 0.9440 \qquad \lim_{\theta\to 0} \kappa = \frac{8}{9} \quad (16)$$

计算得，γ 随 θ 的增大单调增大，κ 随 θ 的增大单调减小。其变化如表1所示。

可见在 θ 取值的大部分范围内，$(\eta P)_{max}$ 态的 P_m 可达 P_{max} 的90%以上，而该态下的循环效率 η_m 在大部分范围约为 P_{max} 态循环效率约1.2倍左右。对热机性能有不同要求时，可设立不同的目标函数进行优化研究。本文的结论表明，在传热规律为 $Q \propto \Delta(\frac{1}{T})$ 时，若从节约能源又不致使输出功率有较大影响的角度考虑，则选择目标函数 $(\eta P)_{max}$ 进行优化设计较为有利。

3.2 与牛顿传热律下有关性能的比较

文献[1]在牛顿传热规律下，假设工质与两热源间的传热系数相同时，求得在 $(\eta P)_{max}$ 态时

$$\eta_m = 1 - \theta\frac{1+\sqrt{1+\frac{8}{\theta}}}{4} \quad (17)$$

当工质与两热源间的传热系数不同时，仍可求得上式。即在牛顿传热规律下，$(\eta P)_{max}$ 态的 η_m 与工质和两热源间的传热系数无关。

表 1　η_m、$\eta_{m\cdot R}$、γ；P_m、P_{max}、κ 随 θ 的变化

θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
η_m	0.7963	0.6762	0.5917	0.5100	0.4310	0.3542	0.2797	0.2072	0.1364	0.0674	0
$\eta_{m\cdot R}$	0.6667	0.5806	0.5000	0.4242	0.3529	0.2857	0.2222	0.1622	0.1053	0.0513	0
γ	1.1458	1.1647	1.1834	1.2023	1.2213	1.2398	1.2588	1.2774	1.2953	1.3138	$1.\dot{3}$
$\frac{T_L}{\alpha}\cdot P_m$	0.1180	0.0864	0.0621	0.0436	0.0296	0.0191	0.0114	0.0060	0.0025	0.0006	0
$\frac{T_L}{\alpha}\cdot P_{max}$	0.1250	0.0920	0.0667	0.0471	0.0321	0.0208	0.0125	0.0066	0.0028	0.0007	0
κ	0.9440	0.9391	0.9310	0.9253	0.9221	0.9183	0.9095	0.9091	0.8984	0.8936	0.8889

注：$\theta=1$ 的 γ 及 κ 是求极限得到的

但式(7)表明，在 $Q\propto\Delta(\frac{1}{T})$ 的传热规律下，η_m 与传热系数之比有关。这说明合理调制传热系数之比在 $(\eta P)_{max}$ 态下，是一个应该特别注意的问题。文献〔2〕的研究表明，Vos 和 Orlov 在研究 $Q\propto\Delta(\frac{1}{T})$ 传热规律下的最大功率 P_{max} 和效率 $\eta_{m\cdot R}$ 时，忽视了传热系数的影响，以致其结果失去了普遍意义。本文结论同样给出，在 $Q\propto\Delta(\frac{1}{T})$ 的传热规律下，对 $(\eta P)_{max}$ 态的 P_m 及 η_m 传热系数的影响同样不可忽视。

3.3　η_m 随 η_c 及 δ 的变化关系

在三种特殊情况下有

$$\lim_{\delta\to 1}\eta_m=3\left(1-\sqrt{1-\frac{4}{9}\eta_c}\right)\qquad(18)$$

$$\lim_{\delta\to\infty}\eta_m=\frac{3}{2}\left(1-\sqrt{1-\frac{8}{9}\eta_c}\right)\qquad(19)$$

$$\lim_{\delta\to 0}\eta_m=\frac{2}{3}\eta_c\qquad(20)$$

其中 $\delta\to\infty$ 相当于 $\alpha\to\infty$，β 有限；$\delta\to 0$ 为 α 有限，$\beta\to\infty$。

分析式(7)以及就以上三式的计算(表 2)均得出，对于任意给定的 δ，η_m 总是随 η_c 的增大而增大。这说明，对于 $(\eta P)_{max}$ 态如同可逆卡诺循环一样，增大高、低温热源的温度差仍然是提高循环效率的途径之一。计算还表明(表 2)，对于任意给定的 η_c，总有

$$\lim_{\delta\to 0}\eta_m<\lim_{\delta\to 1}\eta_m<\lim_{\delta\to\infty}\eta_m\qquad(21)$$

上式指出，增大 δ 是 $(\eta P)_{max}$ 态下 η_m 的又一优化途径。因为按循环效率的基本公式，η_m 同时满足下式

$$\eta_m=1-\frac{Q_2}{Q_1}=1-\frac{\beta(\frac{1}{T_L}-\frac{1}{T_2})t_2}{\alpha(\frac{1}{T_1}-\frac{1}{T_H})t_1}$$

$$=1-\frac{1}{\delta^2}\frac{(\frac{1}{T_L}-\frac{1}{T_2})t_2}{(\frac{1}{T}-\frac{1}{T_H})t_1}\qquad(22)$$

上式定性说明，η_m 随 δ 的增大而增大。

表 2　η_m 对 η_c、δ 的变化关系

$\lim\eta_m$ 　η_c / $\delta\to$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
0	0.2222	0.3333	0.4114
1	0.2310	0.3543	0.4833
∞	0.2416	0.3821	0.5425

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power plant constitutes a basic task of power plant economics analysis. Based on the newest theory of thermal economics, i. e. "Symbol Exergy Economics" proposed by a Spanish scholar, the authors have set up a model for the exergy economics analysis of an energy system. The use of the model on a home-made 200 MW unit has brought about satisfactory results. Key words: energy system, exergy, model, thermodynamics

导热规律为$Q \propto \Delta(\frac{1}{T})$时卡诺热机的$(\eta P)_{max}$ = $(\eta P)_{max}$ of Carnot Heat Engine in Case of Heat Conduction Law Expressed by $Q \propto \Delta(\frac{1}{T})$[刊,中]/Yuan Duqi, Liu Zongxiu (Baoji Institute of Liberal Arts and Science)//Journal of Engineering for Thermal Energy & Power, 1996, 11(6):360～362

The working state $(\eta p)_{max}$ of Carnot heat engin efficiency and power output is studied in the case of heat conduction law being expressed by $Q \propto \Delta(\frac{1}{T})$ with η_m P_m under that state derived. Conducted is a comparison of these values with the maximum power output(p_{max}) under the same heat conduction law. Key words: heat conduction law, heat engine, performance optimization

流化床扬析速率常数的三相传质模型=Three-phase Mass Transfer Model for the Elutriation Rate Constant in a Fluidized Bed [刊,中]/ Chen Hongwei, Jin Baoshen, Xu Yiqian (Southeastern University)//Journal of Engineering for Thermal Energy & Power.-1996, 11(6):363～368

With the help of a fluidized bed three-phase mass-transfer model set up on the basis of bubble assembly theory a simulation was conducted of the fines concentration distribution in a fluidized bed, an thereby the fines elutriation rate constant K(1/min), an important parameter in the design and operation of the fluidized bed, was calculated and discussed. The calculated values have been found to be in good agreement with test results. Key words: fluidized bed, elutriation rate constnt, model, boiler

锅炉螺纹烟管经验计算式选用问题的探讨=An Exploratory Study on the Selection of an Empirical Formula for Helical-ribbed Tubes of Industrial Boilers [刊,中]/ Xu Shiming Yuan Yi (Dalian University of Science & Technology), // Journal of Engineering for Thermal Energy & Power.-1996, 11(6):369～374

Because of their simple construction ,ease of fabrication and better heat transfer properties helical-ribbed tubes are used more often in industrial boilers and heat exchangers than bare tubes. But, up to now it is not possible to perform a complete numerical solution of the helical-ribbed tube heat exchange and resistance characteristics, their calculation being based mainly on empirical formulas obtained from various kinds of experiments. Because of the difference in facilities and working medium employed for the experiments the empirical formulas obtained will also be different, thus resulting in certain deviations as to their form, applicable scope and calculation results. Consequently, the selected empirical formulas for the thermal calculation of boiler flue gas tubes in case of using helical-ribbed tubes will have a significant effect on the accuracy of the boiler flue gas tube thermal calculation. In ·