# 螺旋板稳定性分析和计算

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〔摘要〕基于经典线弹性稳定理论建立了有定距柱支承螺旋板稳定性分析的力学模型 ,推导了计算临界压力的理论公式并给出了工程算例 ,提出一些对工程设计有益的结论。

关键词　螺旋板　稳定性　临界压力
分类号　TU311. 2

## 0　前言

螺旋板是螺旋板换热器主要受压元件 ,相邻螺旋板之间由一系列定距柱支承。换热器工作时 ,螺旋板要受到高压汽体或液体的作用 ,螺旋板凸面所受压力达到一临界值时 ,它们就会突然失去平衡 ,丧失稳定性 ,发生屈曲 ,螺旋板失稳后其抵抗荷载的能力将大大降低 ,以致造成此结构的突然破坏。因此分析螺旋板的稳定性 ,确定它的临界载荷 ,就成为工程设计中的一个重要问题。非钢性薄壁圆筒或壳体呈受外压时存在稳定性问题 ,而且其稳定性破坏先于强度破坏[1]。外压容器设计时 ,稳定性计算是考虑的主要问题。圆筒、壳体以及带加强筋的圆筒、壳体的稳定性计算在许多文献中都有详述 ,但对于在定距柱支承下的壳体的外压作用下的稳定性问题 ,迄今还没有详细的分析和计算。本文将对此类失稳问题以线弹性稳定理论进行分析计算 ,并结合有限无数值解进行验证。

## 1　力学模型和基本假设

### 1. 1　力学模型

定距柱支承的螺旋板的稳定性问题可简化为支承在一排无抗扭刚度柱子上的连续曲板在侧压作用下的稳定性问题。由于螺旋板的曲率变化不大 ,因此可假设曲板半径为 $R$ 的圆柱壳体如图 1所示。壳体母线的长度比定距柱间距大得多 ,因此可忽略边界效应。只考虑远离边界由固定定距柱支承的壳体的稳定性问题 ,如图 2

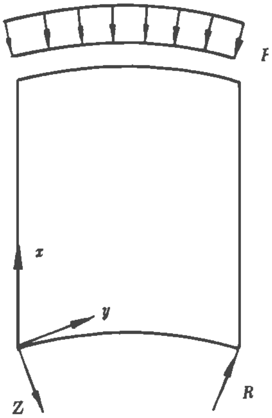

图 1

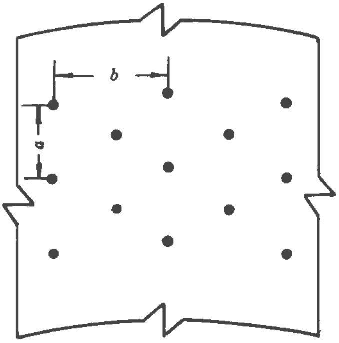

图 2

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**1.2** 本文基本假设

(*a*)失稳前为纯膜力无矩应力状态。

(*b*)薄壳失稳属于弹性范围。

(*c*)壳体受压时其载荷系统是理想的,且不考虑初始缺陷的影响。

## 2 临界压力的计算

薄壳稳定性问题多以非线性方程[2,3]来描述,对于等厚度薄壳其基本方程为:

$$\frac{D}{h}\nabla^2\nabla^2 W = L(W,\mathrm{h}) + K_2^2\frac{\partial^2\mathrm{H}}{\partial x^2} + K_1^2\frac{\partial^2\mathrm{H}}{\partial y^2} + \frac{Z}{h}$$

$$\frac{1}{E}\nabla^2\nabla^2\mathrm{H} = -\frac{1}{2}L(W,W) + K^2\frac{\partial^2 W}{\partial x^2} - K_1\frac{\partial^2 W}{\partial y^2} \qquad (1)$$

式中:

$$L(W,\mathrm{H}) = \frac{\partial^2\mathrm{H}}{\partial y^2}\circ\frac{\partial^2 W}{\partial y^2}\circ\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2\mathrm{H}}{\partial x^2}\circ\frac{\partial^2 W}{\partial y^2} - 2\frac{\partial^2\mathrm{H}}{\partial x\partial y}\circ\frac{\partial^2 W}{\partial x\partial y}$$

$$L(W,W) = 2\left(\frac{\partial^2 W}{\partial x^2}\circ\frac{\partial^2 W}{\partial y^2} - \frac{\partial^2 W}{\partial x\partial y}\right)$$

该式为非线性的八阶微分方程组,在数学处理上非常困难,即使在最简单的条件下,若以常用的幂级数法、摄动法也只能得到近似解。在本文的基本假设条件下屈曲微分方程[4]简化为:

$$\frac{P}{h}\nabla^8 W + \frac{Eh}{R^2}\frac{\partial^4 W}{\partial x^4} + \nabla^4\left(e_1^0\frac{\partial^2 W}{\partial x^2} + 2f^0\frac{\partial^2 W}{\partial x\partial y} + e_2^0\frac{\partial^2 W}{\partial y^2}\right) + \nabla^4 P = 0 \qquad (2)$$

前屈曲状态为无矩应力状态,壳体受均匀侧压力 $P$ 的作用,则

$$e_1^0 = 0 \quad e_2^0 = \frac{PR}{h} \quad f^0 = 0 \qquad (3)$$

上式代入圆柱壳屈曲微分方程式,得到:

$$\frac{D}{h}\nabla^8 W + \frac{E}{R^2}\cdot\frac{\partial^4 W}{\partial x^4} + \frac{PR}{h}\nabla^4\left(\frac{\partial^2 W}{\partial y^2}\right) = 0 \qquad (4)$$

圆柱壳的曲边可以看成简支,因此可设屈曲形状函数为

$$W = \mathrm{EE}_{m\,n} f_{mn}\sin a_m x\sin \mathrm{U}_n y$$

$$(m = 1,2,3,\cdots, n = 1,2,3) \qquad (5)$$

式中: $a_m = \frac{m\mathrm{c}}{L}, \mathrm{U}_n = \frac{n}{R}$

$$D(a_m^2 + \mathrm{U}_n^2)^4 + \frac{Eh}{R^2}a_m^4 - PR(a_m^2 + \mathrm{U}_n^2)^2\frac{n^2}{R^2} = 0$$

设无量纲的载荷为 $\overline{P} = \frac{PR^2}{Eh^2}$,则上式可写成

$$\overline{P} = \frac{1}{12(1-\_^2)}(1+\theta^2)^2\mathrm{Z} + \frac{\theta^4}{(1+\theta^2)\mathrm{Z}} \qquad (6)$$

引进符号:

$$\theta = \frac{L_2}{L_1} = \frac{m\mathrm{c}R}{nL} = \frac{a_m}{\mathrm{U}_m} \qquad (7)$$

$$\mathrm{Z} = \frac{n^2 h}{R} \qquad (8)$$

$$L_2 = \frac{2\mathrm{c}R}{2n}, L_1 = \frac{L}{m} \qquad (9)$$

式中: $L_2$ 和 $L_1$ 分别为屈曲波形沿圆周和沿母线方向上的半波长度;θ为两种波纹形状的半波长之比。

从式(4)可以看出,当θ值增加时,右端两项均增加,对临界压力而言,θ应取最小值。无定距柱时应取 $m = 1$,而有定距柱时,受定距柱的影响,在相邻定距柱之间发生局部失稳时,沿轴向在相邻定距柱之间至少形成一个半波,这样取 $m = L/a$。于是:

$$\theta = \frac{\mathrm{c}R}{a}\cdot\frac{1}{n} = \frac{1}{n} \qquad (10)$$

其中 $A = \mathrm{c}R/a$

对临界压力而言,应选取 $n$ 使临界力达到最小值。由于受定距柱的影响,发生局部失稳时沿周向在相邻定距柱之间也至少形成一个半波,于是周向半波数

$$n = \frac{\mathrm{c}R}{b}K = BK \quad K = 1,2,3,\cdots \qquad (11)$$

其中 $B = \mathrm{c}R/b$ 为无量纲系数,$K$ 为沿周向相邻定矩柱之间的半波数。

$$\overline{P} = \frac{1}{12(1-\_^2)}\cdot\frac{\lambda^2+K^2}{K^2}\cdot G + \frac{\gamma^4}{(\gamma^2+\mathrm{K}^2)^2K^2}\cdot\frac{1}{G} \qquad (12)$$

其中 $\lambda = A/B = b/a$, $G = B^2C = \mathrm{c}^2Rh/b^2$ (无量纲参数)。

λ是沿周向与轴向定距柱间距之比,与定距柱排列方式有关。取不同的λ值,可以得到 $\overline{P}$ 与 $G, K$ 的曲线,图 3是 $\lambda = \overline{3}$ 时,$\overline{P}$ 与 $G, K$ 的曲线,图中根据 $G$ 值的大小,确定 $K$ 的取值使临界边达到最小值。

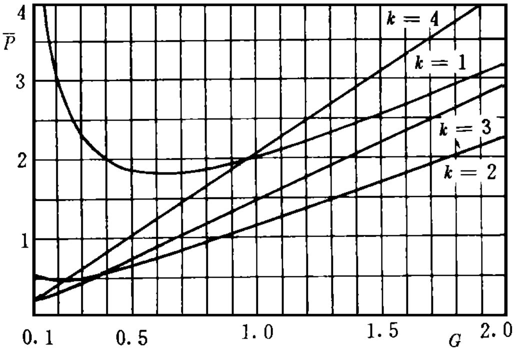

图 3　$\overline{P}$与 $G$ $K$关系曲线

## 3 算例

螺旋板换热器四个定距柱组成的菱形短对角线沿轴向,螺旋板厚 $h=3$ mm,螺旋板最大半径 $R$分别为 300,400,500 mm,定距柱间距 $a$分别为 10,100,150,200,250,300 mm。半径 $R$取不同值,定距柱间距也取不同值时,分别计算 $G$值,根据 $G$值,确定 $K$的取值,计算结果列于表 1中。

表 1

| | | | | | | |
|---|---|---|---|---|---|---|
| $R=300$ mm | | | | | | |
| 定距柱间距 $a$(mm) | 50 | 100 | 150 | 200 | 250 | 300 |
| $G$ | 1.184 | 0.296 | 0.132 | 0.074 | 0.047 | 0.033 |
| $K$ | 2 | 3 | 3 | 4 | 4 | 5 |
| 无量纲临界压力 $\overline{P}$ | 1.367 | 0.457 | 0.246 | 0.174 | 0.131 | 0.108 |
| 临界压力 $P_{cr}$(MPa) | 27.35 | 9.15 | 4.91 | 3.48 | 2.62 | 2.17 |
| 有限元数值解(MPa) | 25.95 | 8.65 | 4.80 | 3.36 | 2.54 | 2.09 |
| 误差(%) | 5.10 | 3.30 | 2.24 | 3.45 | 3.05 | 3.69 |
| $R=400$ mm | | | | | | |
| $G$ | 1.579 | 0.395 | 0.175 | 0.099 | 0.063 | 0.044 |
| $K$ | 2 | 2 | 3 | 3 | 4 | 4 |
| 无量纲临界压力 $\overline{P}$ | 1.801 | 0.559 | 0.297 | 0.215 | 0.155 | 0.126 |
| 临界压力 $P_{cr}$(MPa) | 20.26 | 6.29 | 3.34 | 2.42 | 1.75 | 1.42 |
| 有限元数值解(MPa) | 19.2 | 6.02 | 3.25 | 2.36 | 1.68 | 1.36 |
| 误差(%) | 5.23 | 4.29 | 2.69 | 2.48 | 4.0 | 4.2 |
| $R=500$ mm | | | | | | |
| $G$ | 1.974 | 0.494 | 0.219 | 0.123 | 0.079 | 0.055 |
| $K$ | 2 | 2 | 3 | 3 | 4 | 4 |
| 无量纲临界压力 $\overline{P}$ | 2.238 | 0.647 | 0.353 | 0.237 | 0.183 | 0.142 |
| 临界压力 $P_{cr}$(MPa) | 16.11 | 4.66 | 2.54 | 1.71 | 1.32 | 1.02 |
| 有限元数值解(MPa) | 15.2 | 4.45 | 2.45 | 1.66 | 1.29 | 0.98 |
| 误差(%) | 5.65 | 4.51 | 3.54 | 2.92 | 2.27 | 3.92 |

使用通用有限元屈曲分析(稳定性分析)程序对以上各种情况进行计算,计算结果见表 1。由表 1可见,本文公式解与有限元数值解基本吻合。

使用线弹性通用有限元分析程序对不同半径、不同定距柱间距情况下的螺旋板进行应力分析,确定发生强度破坏的压力值。其计算结果与临界压力的对比见图 4图 5图 6

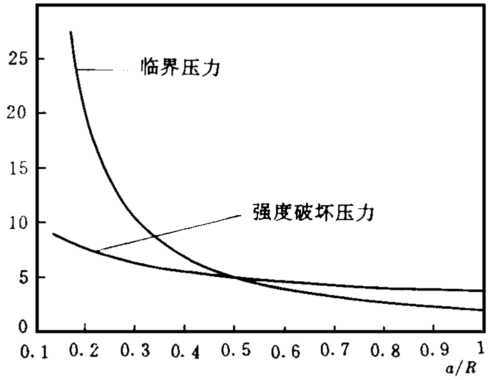

图 4　P与 a /R关系曲线 (R= 300 mm)

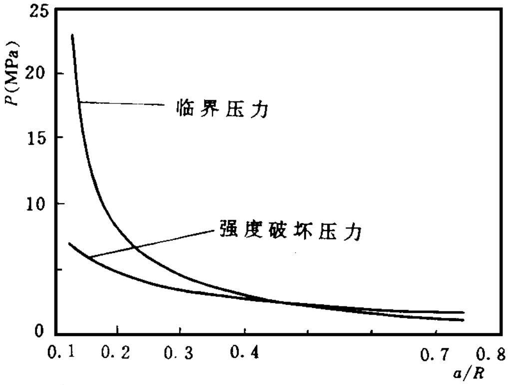

图 5　P与 a /R关系曲线 (R= 400 mm)

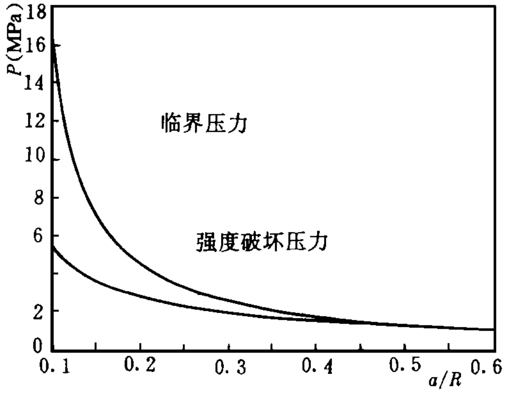

图 6　P与 a /R关系曲线 (R= 500 mm)

由图 4到图 6的临界压力与强度破坏压力对比可见,失稳破坏和强度破坏的先后与定距柱间距 a和半径 R之比 a /R有关,我们发现 a /R> 0.5时螺旋板失稳破坏在先,反之,a /R < 0.5时,螺旋板强度破坏在先。

## 4　结论

(1)本文临界压力计算公式得到的理论解与有限元数值解基本吻合,表明力学模型和公式推导是可靠的。

(2)定距柱间距对在侧压作用下有定距柱支承的壳体、螺旋板的临界压力值影响较大。a /R> 0.5时,螺旋板失稳破坏在先;反之,a /R < 0.5时,螺旋板强度破坏在先,因此在设计时根据 a /R之值选择设计准则。

### 符号说明

a— 轴向相邻定距柱间距,(mm);
b— 周向相邻定距柱间距,(mm);
D— 弯曲刚度,$D= Eh^3/12(1-\_^2)$;
E— 弹性模量,(MPa);
G— 无量纲参数;
h— 螺旋板厚度,(mm);
L— 螺旋板宽度,(mm);
$1_1$— 失稳时,沿轴向的半波长,(mm);
$1_2$— 失稳时,沿周向的半波长,(mm);
M— 稳定性安全系数;
m— 失稳时,沿周向的半波数;
n— 失稳时,沿周向的半波数;
P— 设计压力,(MPa);
$P_{cr}$— 临界压力,(MPa);
$\overline{P}$— 无量纲载荷;
R— 螺旋板曲率半径,(mm);
θ— 周向半波长与轴向半波长之比,$\theta= 1_2/1_1$
$= m^{C}R/nL$;
λ— 周向与轴向定距柱间距之比;
μ— 材料的泊桑系数。

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enhanced circulating rate of particles and the decrease in particle diameter will be beneficial to the gas/solid interphase heat transfer. Also obtained is a corresponding dimensionless equation. **Key words**: circulating fluidized bed, gas phase, solid phase, heat transfer

隔膜式气压给水设备的节能研究 = **A Study on the Energy-saving of Diaphragm Air-pressure Water Supply Installation**〔刊,中〕/Wu Xifu (Zhejiang Industrial University) //Journal of Engineering for Thermal Energy & Power. - 1997, 12(4). - 289~291

On the basis of energy consumption test of a diaphragm variable-pressure and constant-pressure water supply installation energy-saving analyses are performed with respect to the air feeding of air compressors, the utilization of exhaust gas, the selection of water pumps and the number of times of water pump startups. Energy-saving measures are proposed. All the above can serve as a guide for design and operation management. **Key words**: diaphragm type, water supply installation, energy-saving

复杂换热器系统的动态特性计算 = **Calculation of the Dynamic Characteristics of a Complex Heat Exchanger System**〔刊,中〕/Li Zheng, Sun Xin, Ni Weidou (Tsinghua University) //Journal of Engineering for Thermal Energy & Power. - 1997, 12(4). - 292~296

This paper deals with a method for calculating the dynamic characteristics of a complex heat exchanger system, which was developed during the modelling and simulation of a 220 t/h home-made CFB boiler. Based on an approximate analytical solution the said method significantly enhances the calculation speed and solves the problem of non-convergence during calculations. A decoupling method has been adopted, Which separates the balance calculation of cold and hot working mediums, making it possible to completely avoid iterative computations. By combining the approximate analytical solutions and the decoupling calculation method established is a set of generalized method for calculating the dynamic behavior of heat exchanger systems, thereby providing a general-purpose, high-efficient and simple calculation method for complex heat exchangers. **Key words**: natural circulation boiler, heat exchanger, dynamic characteristics, calculation

再热抽汽式汽轮机中压缸末级叶片压差保护控制 = **Differential-pressure Security Control of the Last-stage Blades of a Reheat Extraction Steam Turbine Intermediate-pressure Cylinder**〔刊,中〕/Yu Daren, Wang Xitian, et al (Harbin Institute of Technology) //Journal of Engineering for Thermal Energy & Power. - 1997, 12(4). - 297~299

Discussed in this paper is the differential-pressure security control of the last-stage blades of a reheat extraction steam turbine intermediate-pressure cylinder. Through an analysis of the simulation results of a mathematical model basic measures for the blade differential-pressure security control have been summed up. **Key words**: reheat extraction steam turbine, blade, differential-pressure security control

螺旋板稳定性分析和计算 = **The Analysis and Calculation of a Spiral Plate Stability**〔刊,中〕/Zhou Chuanyue, et al. (Harbin No. 703 Research Institute) //Journal of Engineering for Thermal Energy & Power. - 1997, 12(4)-300~303

Based on a classical linear elastic stability theory the authors have set up a mechanics model for stability analysis of a spacing column-supported spiral plate. A theoretical formula for calculating the theoretical pressure of a critical pressure has been derived with some engineering calculation examples being presented. Some conclusions helpful for performing engineering design are also proposed. **Key words**: spiral plate, stability, critical pressure

综合似然率(GLR)试验在传感器故障检测中的应用 = Application of a Generalized Likelihood Ratio (GLR) Test in Sensor Failure Detection〔刊, 中〕/Huang Shanheng, Zhu Qiaobin, et al (Shanghai Jiaotong University) //Journal of Engineering for Thermal Energy & Power. - 1997, 12(4). - 304~306