

# 弹性转子磁气轴承系统的 $H_\infty$ 控制

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**摘要:** 根据有限元法, 导出了弹性转子低阶设计模型, 针对模型中低阶刚性模式和弹性一阶模式, 提出一种考虑  $H_\infty$  控制的鲁棒稳定性和混合灵敏度设计方法。仿真结果表明, 对含有弹性模式的低阶模型控制具有良好的稳定性和高衰减度, 并对模型变参数工况具有较强的鲁棒性。

**关键词:** 弹性转子; 磁气轴承;  $H_\infty$  控制; 鲁棒稳定性; 混合灵敏度

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## 1 引言

目前国内外在旋转机械支撑物的研究领域中, 对磁气轴承旋转机械的研究十分活跃。在弹性转子采用磁气轴承情况下, 若以常规方法设计控制器, 则系统稳定性将受到影响, 而采用对模型不确定性具有良好控制效果的  $H_\infty$  鲁棒控制理论, 在低阶模型控制系统<sup>[1]</sup>、振动控制系统<sup>[2]</sup>及磁气轴承控制系统<sup>[3,4,6]</sup>的设计等方面已取得一些研究成果。

本文研究中把作为连续体的弹性转子划分为低阶刚性模式和弹性一阶模式两个低阶模型, 从  $H_\infty$  控制的鲁棒稳定性和混合灵敏度两方面进行了控制系统设计和方案比较, 并针对刚性模式设计了具有积分特性的控制器。结果表明, 以  $H_\infty$  混合灵敏度控制法设计的控制器, 对低阶模型对

象的控制具有良好的稳定性和高衰减度, 同时对模型变参数工况, 具有较强的鲁棒性。

## 2 模型的建立

### 2.1 弹性转子磁气轴承系统模型的建立

图1所示为弹性转子磁气轴承系统的径向截面图, 弹性转子两端为磁气轴承, 根据有限元法, 建立图1(a)所示弹性转子模型运动方程式:

$$M_0 \ddot{q} + K_0 q = 0 \quad (1)$$

式中:  $q = [x_1 \ \theta_1 \ x_2 \ \theta_2 \ x_3 \ \theta_3]^T$ ,  $M_0$  为质量矩阵,  $K_0$  为刚性矩阵。

图1(b)所示磁气轴承电磁石模型满足下式:

$$V = L \frac{dI}{dt} + RI \quad (2)$$

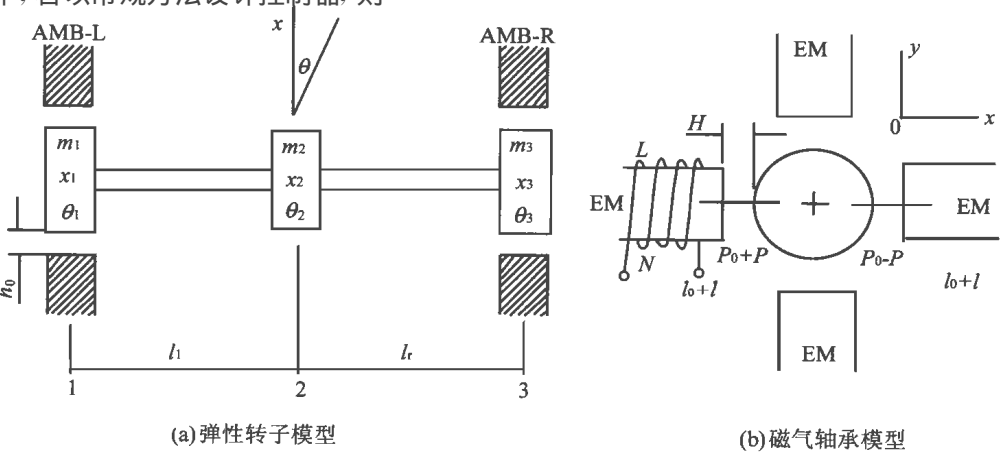


图1 弹性转子磁气轴承系统模型

式中:  $V$ : 线圈输入电压,  $L$ : 线圈电感,  $R$ : 线圈电阻,  $I$ : 线圈电流。

在平衡点附近微小振动情况下, 有下式成立:

$$P = p_0 + p, I = i_0 + i, H = h_0 + h \quad (3)$$

式中:  $p_0$ 、 $i_0$ 、 $h_0$  分别为额定吸引力、电流、间隙长度,  $p$ 、 $i$ 、 $h$  分别为偏移控制吸引力、电流、间隙长

度。将式(3)关于  $i, h$  作泰勒展开(其中  $i_0 \gg i, h_0 \gg h$ ), 且仅取线性项, 则控制吸引力为:

$$P = 2p_0(\frac{i}{i_0} - \frac{h}{h_0}) = 2p_0 \frac{i}{i_0} - 2p_0 \frac{h}{h_0} \quad (4)$$

等式右边第一项为控制吸引力, 第二项为偏移吸引力。因式(1)的弹性转子被式(4)的控制吸引力约束, 则式(1)可修改为:

$$M_0 \ddot{q} + K_0 q = FP \quad (5)$$

式中:  $F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T, P = \begin{bmatrix} P_l \\ P_r \end{bmatrix}, P_l = -4p_0$

$(\frac{i_l}{i_0} - \frac{x_1}{h_0});$

$P_r = -4p_0(\frac{i_r}{i_0} - \frac{x_3}{h_0})$ 。考虑额定电压和电流的关系, 可得系统的状态方程:

$$\dot{x}_f = A_f x_f + B_f u \quad (6)$$

式中:  $x_f = [\xi \ \dot{\xi} \ i]^T, u = [u_l \ u_r]^T,$

$$A_f = \begin{bmatrix} 0 & I & 0 \\ -\Omega^2 & -2\xi\Omega & \Phi^T F_i \\ 0 & 0 & E_i \end{bmatrix}, \Phi^T M \Phi = I,$$

$$F_i = \begin{bmatrix} -4\frac{P_0}{i_0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4\frac{P_0}{i_0} & 0 \end{bmatrix}^T,$$

$$E_i = \begin{bmatrix} -\frac{R}{L} & 0 \\ 0 & -\frac{R}{L} \end{bmatrix}, B_f = [0 \ 0 \ E_v]^T,$$

$$E_v = \begin{bmatrix} -\frac{1}{L} & 0 \\ 0 & -\frac{1}{L} \end{bmatrix}。根据测得的转子位移, 可得磁$$

气轴承系统的输出方程:

$$y = C_f x_f = [x_1 \ x_3]^T \quad (7)$$

### 2.2 系统低阶化模型的建立

由于弹性转子中刚性模式的存在, 使得系统自身存在不稳定性, 而弹性模式虽然衰减性差, 但其本质是稳定的。因此, 在建立系统低阶模型时, 要考虑刚性模式的稳定性和弹性模式的可控阶次等问题。下面是系统  $K$  阶低阶模型状态方程:

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r u + D_r w \\ y &= C_r x_r \end{aligned} \quad (8)$$

对于式(8)描述的低阶模型, 进行两种控制系统的设计:

- (1)  $K = 2$ , 刚性模式的控制;
- (2)  $K = 3$ , 刚性模式和弹性一阶模式的控制

### 3 $H_\infty$ 控制系统的设计

对于式(8)的低阶模型对象, 具有  $H_\infty$  控制器的闭环控制系统如图 2 所示, 图中,  $A_r, B_r, C_r$  为低阶

控制对象,  $u$  为控制输入,  $y$  为实现观测输出,  $Z_1, Z_2$  为控制量,  $W_1(s), W_2(s)$  为加权函数矩阵。  $H(s)$  控制器设计为下式:

$$H(S) = C_H(SI - A_H)^{-1} B_H \quad (9)$$

其中,  $H(S)$  对低阶模型及全阶模型的闭环控制系统鲁棒稳定, 且对  $K$  阶模型的控制具有高的衰减率。

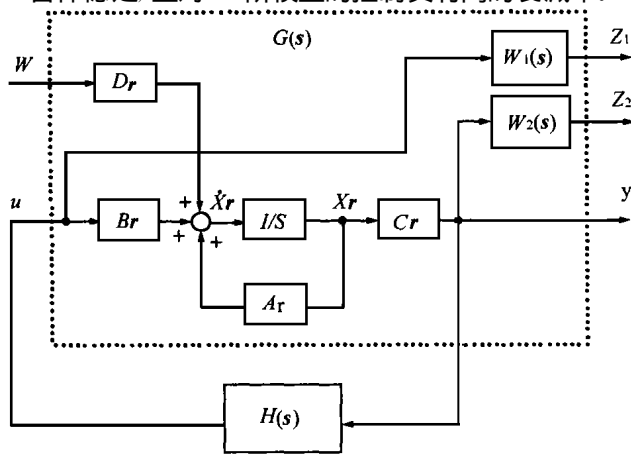


图 2  $H_\infty$  控制器闭环系统框图

下面首先考虑鲁棒稳定性的设计, 图 2 中令  $W_2(S) = 0$ , 有下式:

$$T(S) = H(S)[I - P_r(S)H(S)]^{-1} P_r(S) \quad (10)$$

式中:  $P_r(S) = C_r(SI - A_r)^{-1} B_r; T(S)$  为互补灵敏度函数, 根据对象模型误差评价系统的稳定裕度。当模型误差存在时, 鲁棒稳定的条件是  $T(S)$  的  $H_\infty$  范数尽可能的小, 可由小增益定理定义为下式:

$$\| \Delta(S) T(S) \|_\infty < 1 \quad (11)$$

以满足下式的加权函数矩阵  $W_1(S)$ , 确定误差  $\Delta(S)$  的上限:

$$\bar{\sigma}[\Delta(j\omega)] \leq W_1(S) \quad (12)$$

对于满足式(12)的所有  $\Delta(j\omega)$ , 闭环系统鲁棒稳定的必要充分条件是:

- (1)  $H_\infty$  控制器  $H(S)$  使  $P_r(S)$  稳定;
- (2)  $\| W_1(S) T(S) \|_\infty < 1$ 。

图 2 中当灵敏度降低时, 可考虑混合灵敏度的设计为下面形式:

$$\left| \begin{matrix} W_1(S) T(S) \\ W_2(S) M(S) \end{matrix} \right|_\infty < 1 \quad (13)$$

式中:  $M(S) = [I - P_r(S)H(S)]^{-1} P_r(S)$  称为调整函数。若把  $M(S)$  在低频域内极小化, 使其复平面上的极点远离虚轴向左半面移动, 则控制器具有良好的稳定性和快速收敛性。

### 4 仿真分析

附表为系统模型设计参数, 仿真模型包含刚性和弹性 1 阶模式。图 3 为弹性转子稳定上浮时, 在中央质量盘上有 0.5kg 的不均衡力作用下, 系统的位移输出响应。结果表明, 鲁棒稳定性法能使不稳定的

刚性模式稳定,但不能保证弹性模式的稳定性,而采用带通型加权函数  $W_2(S)$  根据混合灵敏度法设计的控制器,可使弹性 1 阶模式的衰减率从控制前的  $1/1000$  变为控制后的  $1/27$ , 振动被有效的控制。

附表 仿真模型参数

参 数	值	单 位
质量	$m_1$	1.50 kg
	$m_2$	0.80 kg
	$m_3$	1.50 kg
长度	$l_1 = l_r$	0.50 m
直径	$d$	0.02 m
衰减率	$\xi$	0.001 m
间隙	$h_0$	0.001 m
额定电流	$i_0$	1.0 A
额定吸引力	$P_0$	50 N
线圈电感	$L$	0.33 H
线圈阻抗	$R$	23.6 $\Omega$
弹性模式	$\omega_1$	40.9 Hz
频率	$\omega_2$	181.5 Hz
	$\omega_3$	389.5 Hz
	$\omega_4$	787.3 Hz

各质量点上浮的动态响应。比较图 4(a)、(b)、(c) 可知,系统的特性几乎没有发生变化,从而表明  $H_\infty$  控制对于模型变参数工况具有较强的鲁棒性。

### 5 结束语

在弹性转子磁气轴承控制系统中,应用  $H_\infty$  控制理论,从鲁棒稳定性和混合灵敏度两方面进行了控制系统设计和仿真,结果如下:

(1)对于具有多种模式的弹性转子磁气轴承系统的低阶模型,应用  $H_\infty$  控制理论设计的控制器,可使系统实现稳定的上浮。

(2)对于含刚性模式的低阶模型,以鲁棒稳定性法设计的  $H_\infty$  控制器没有积分特性。

(3)对于含刚性模式的低阶模型,以混合灵敏度法设计的  $H_\infty$  控制器和常规 PID 控制器具有相同的积分特性。

(4)

对于含弹性模式的低阶模型,以混合灵敏度法设计的  $H_\infty$  控制器,可有效地控制系统的振动。

(5)为了防止系统发散和克服模型参数变动的影 响,作为弹性转子磁气轴承系统的控制方法,  $H_\infty$  控制是一种有效的方法。

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(渠 源 编 辑)

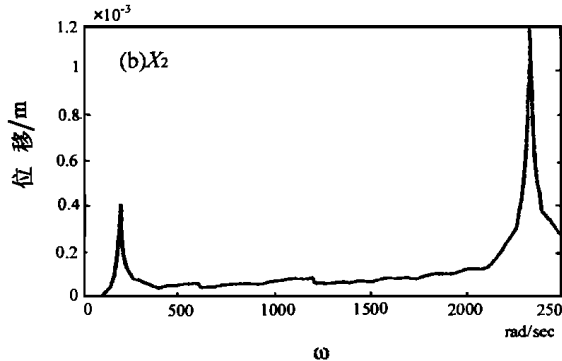
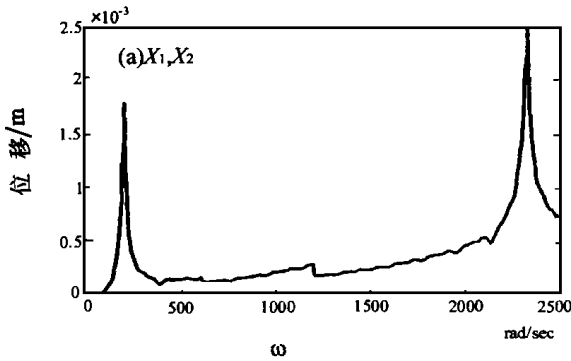


图 3 弹性转子受不均衡力作用的频率响应

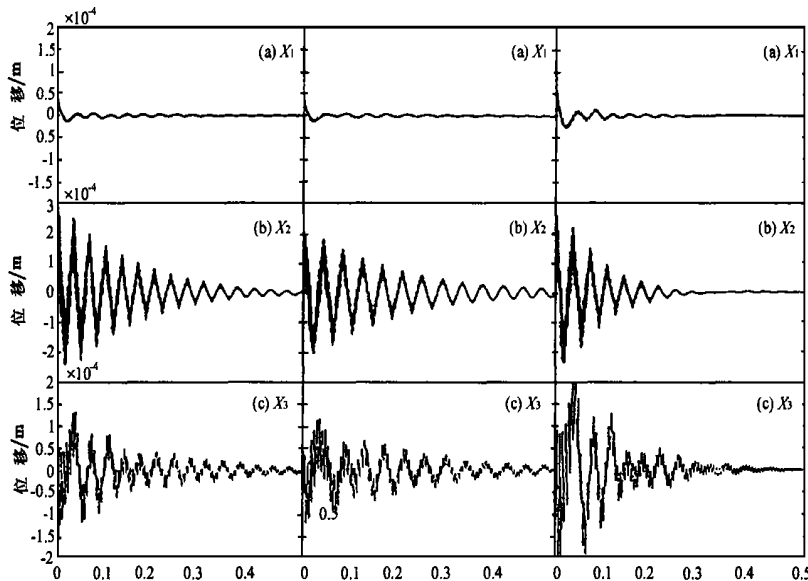


图 4 质量点  $x_1, x_2, x_3$  上浮动态响应

图 4(a) 是各质量点上浮时的初始值响应(初值为:  $x_1 = 0.001m, x_2 = x_3 = 0$ ); 图 4(b) 是中央质量点质量增加 2 倍时, 各质量点上浮的动态响应; 图 4(c) 是当磁气轴承的吸引力偏移设定值 50% 时,

ing method for the system can reduce the influence of quantization error of A/D converter, thereby increasing the measurement accuracy of the temperature-rise rate and enhancing the accuracy of thermal stress calculation. In addition, the system on the basis of a measured thermal stress gives an output in the form of 4 - 20 mA to other systems for analysis, accumulating relevant data for computing turbine service life later on. **Key words:** rotor thermal stress, real-time monitoring, difference measuring method, accuracy

稠密气固两相流的直接数值模拟 = **Direct Numerical Simulation of Dense Gas-solid Two-phase Flows** [刊, 中] / Yuan Zhulin (Thermal Energy Research Institute under the Southeastern University) // Journal of Engineering for Thermal Energy & Power. — 1999, 14(6). — 465 ~ 466

Gas-phase field and discrete particle field are treated respectively by a Eulerian method and a Lagrangian one. During the treatment of a particle field the effect of particle diameter, specific weight, rigidity of material and friction factor, etc on particle movement has been taken into account. A direct simulation method was employed to simulate funnel flow, the particle movement in a ball mill and a stouted bed. Moreover, tests were conducted to verify the simulation results obtained on the stouted bed. **Key words:** gas-solid two-phase flow, direct numerical simulation

换热系统变工况分析 = **Off-design Performance Analysis of a Heat Exchange System** [刊, 中] / Bao Demei, Fan Deshan, Xu Zhigao (Southeastern University) // Journal of Engineering for Thermal Energy & Power. — 1999, 14(6). — 467 ~ 470

A new method for analyzing a heat exchange system performance variation is proposed along with the establishment of a relevant linear mathematical model. The proposed method can not only analyze the performance of the heat exchange system as a whole during a change in operating conditions but also reflect the thermal excursion and temperature changes of each heat exchanger within the system and also the efficiency of the heat exchanger itself. Finally, by taking the boiler heating surface soot-blowing as an example the results obtained from the model and those from a simulated model are compared. It is shown that the proposed method features both simplicity and real-time properties. **Key words:** heat exchange system, off-design operating conditions, thermal efficiency, heat transfer unit, soot-blowing

基于模糊神经网络的高加系统内部故障诊断方法 = **A Method for the Diagnosis of Internal Malfunctions of a High-pressure Heater System Based on a Fuzzy Neural Network** [刊, 中] / Qin Zaicong, Xu Zhigao (Southeastern University), Lu Songlin (Jiangsu Provincial Electrical Power Test Research Institute) // Journal of Engineering for Thermal Energy & Power. — 1999, 14(6). — 471 ~ 472

The authors expound the application of a fuzzy neural network for the diagnosis of internal malfunctions in a high-pressure heater system. Practice has shown that the diagnosis model under discussion has broad prospects for engineering applications. **Key words:** failure diagnosis, fuzzy neural network, high-pressure heater system

双列调节级的变工况热力计算方法及应用 = **A Method of Thermodynamic Calculation for Off-design Conditions of a Turbine Dual-row Governing Stage and Its Application** [刊, 中] / Fu Lin, Jiang Yi (Qinghua University) // Journal of Engineering for Thermal Energy & Power. — 1999, 14(6). — 473 ~ 476

The authors have come up with a thermodynamic calculation method for a turbine dual-row governing stage. Under this method the thermodynamic properties of the governing stage, including post-stage steam enthalpy, can be speedily identified when made known are only such parameters as the relevant geometric characteristics of the stage. The method can be employed for the simplified thermodynamic calculation of heat supply units. **Key words:** dual-row governing stage, algorithm, steam extraction unit

矩阵法和偏微分理论在机组热经济性分析中的应用 = **The Use of Matrix Method and Partial Differential Theory for the Analysis of a Reheat Unit Economic Performance** [刊, 中] / Zheng Xiuping, Zheng Luying, Cai Tianyou (Northeastern University) // Journal of Engineering for Thermal Energy & Power. — 1999, 14(6). — 477 ~ 480

A general analysis is performed of a power plant reheat-regeneration thermodynamic system with the use of a matrix method and partial differential theory. Given are the calculation results of thermo-economic analytical parameters  $H_j^0$  and  $\eta_j^0$ . The proposed method is applicable for both reheat units and non-reheat ones. **Key words:** thermal system, matrix method, partial differential theory, economic performance analysis

弹性转子磁气轴承系统的  $H_\infty$  控制 =  **$H_\infty$  Control of the Magnetic Bearing System of a Flexible Rotor** [刊,

中] / Li Dazhong (North China University of Electric Power Engineering) // Journal of Engineering for Thermal Energy & Power. — 1991, 14(6). — 481 ~ 483

Through the use of finite element method this paper deduces a low-order design model for a flexible rotor. With regard to the low-order rigidity mode and the flexible one-order mode in the above-cited model the author has proposed a design method, in which the robustness stability and mixed sensitivity of the  $H_\infty$  control are taken into account. The simulation results show that the proposed design method features a good stability and high attenuation for the flexible mode-containing low-order model control. Furthermore, under this method a fairly good robustness to the parameter variation of the model has also been attained. **Key words:** flexible rotor, magnetic bearing,  $H_\infty$  control, robustness stability, mixed sensitivity

汽轮发电机组振动故障的多征兆诊断方法 = **Vibration Failure Multiple-symptom Diagnosis Method for Turbogenerator Sets** [刊, 中] / Li Yong (Northeast China Institute of Electrical Power Engineering), Sun Haibo (Beijing Electrical Power Engineering College), Ca Zuqing (Southeastern China University) // Journal of Engineering for Thermal Energy & Power. — 1999, 14(6). — 484 ~ 486

Presented in this paper is a multiple-symptom diagnosis method for assessing vibration failures of turbogenerator sets. Moreover, to solve the problem of recurring different diagnosis results when vertical and horizontal vibration signals are used, a comprehensive diagnosis method is proposed, which can integrate vertical and horizontal vibration signals. **Key words:** turbogenerator set, vibration, failure diagnosis

汽轮发电机组轴心轨迹特征的自动提取及辨识 = **The Automatic Extraction and Identification of the Shaft Centerline Locus Characteristics of a Turbogenerator Set** [刊, 中] / Zhang Xinjiang, et al (Harbin Institute of Technology) // Journal of Engineering for Thermal Energy & Power. — 1999, 14(6). — 487 ~ 488

On the basis of the specific features of a turbogenerator rotor shaft centerline locus and by utilizing the totally different character of the variation with scale of wavelet transformation modulus maximum value of signal and noise to eliminate noise, the authors conducted an extraction of shaft centerline locus. Moreover, a new type of variable equi-length compression coding of plane graphics is proposed. With the help of this method it is possible to achieve a relatively sizable compression of the shaft centerline locus graphics coding after the noise abatement. This has led to a decrease in output of the identification system of the shaft centerline locus neural network, thus increasing the network training speed. Meanwhile, the accuracy and stability of the neural network identification system undergo a simultaneous enhancement. **Key words:** turboogenerator set, shaft centerline locus, wavelet transformation, neural network

抛煤机链条炉内控制与消烟除尘 = **Furnace Control of a Spreader and Chain Grate Stoker Boiler and Related Smoke Abatement and Dust Removal** [刊, 中] / Lu Wenjin, Peng Dongyou (Sichuan Resin Factory) // Journal of Engineering for Thermal Energy & Power. — 1999, 14(6). — 489 ~ 490

Spreader and chain grate stoker boilers generally suffer from a low thermal efficiency and steam output as well as serious environmental pollution caused by excessive smoke emissions. In view of this, the authors have in the light of certain specific conditions analyzed the boiler furnace combustion mechanism and aerodynamics and presented some measures and schemes for improving boiler furnace control to rectify the above-cited deficiencies.

**Key words:** spreader and chain grate stoker boiler, combustion mechanism, circulating combustion, smoke abatement and dust removal

调整危急保安器动作转速的一种简捷方法 = **A Simplified and Speedy Method for Adjusting the Actuation Speed of Emergency Tripping Devices** [刊, 中] / Xing Hui (Shandong Binzhou Chemical Group Co. Ltd.) // Journal of Engineering for Thermal Energy & Power. — 1999, 14(6). — 491 ~ 492