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用时间相关法求解定常粘性流场的加速收敛法

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摘　要:作者详细分析了显、隐格式的CFL数后,指出影响CFL数大小的关键原因在于离散方式。要提高用时间相关法求解定常流场的收敛速度,就必须加大CFL数。而加大CFL数的最直接、最有效的方法是选用隐式方法。作者在兼顾CFL数和精度的情况下,用Taylor展开法结合TVD格式给出了一类加速收敛的隐式方法。

关　键　词:TVD格式;隐式方法;加速收敛;粘性流场

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1　引言

时间相关法由于将定常和非定常问题统一地用一组双曲型的偏微分方程组来描述,类型确定,算法易于设计,从而自70年代以来,倍受人们的关注。

时间相关法分显式推进和隐式推进两种。对定常问题,加速收敛的意义在于取定初场后,经较少的时间步使流场收敛趋于定常,这时对加速收敛起决定作用的因素有两个,一个是加大CFL数,另一个是格式本身的精度。下面我们就模型方程分析一下影响显、隐格式CFL数大小的因素。

为说明问题方便,以最简单的模型方程为例进行讨论

$$\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}=0,\ a>0 \tag{1}$$

将式(1)显示离散为

$$\left(\frac{\partial u}{\partial t}\right)_j^{n+1}+a\left(\frac{\partial u}{\partial x}\right)_j^{n}=0,\ a>0 \tag{2}$$

对式(2)中的差分取一阶迎风差分有

$$u_j^{n+1}=u_j^n-a\lambda(u_j^n-u_{j-1}^n),\quad \lambda=\frac{\Delta t}{\Delta x} \tag{3}$$

式(3)是一个显式格式,其稳定性条件为$a\lambda\leqslant 1$,当$a\lambda=1$时,$u_j^{n+1}=u_{j-1}^n$恰为微分方程(1)的精确解。稳定性条件说明,$n+1$层不能任取,即时间步长Δt需要满足关系式$a\frac{\Delta t}{\Delta x}\leqslant 1$,究其原因,将式(1)离散为式(2)时是有条件的,即式(2)作为式(1)的近似方程的条件是稳定性条件,超出这个条件时,式(2)就不能作为式(1)的近似方程。这说明显式离散在加大稳定性时间步长方面有一个不可逾越的障碍——CFL数必须小于等于1。实际应用时,人们为了加大CFL数增大时间步长Δt,常采用Runge-Kutta方法来加速收敛,令$R=\left[-a\frac{\partial u}{\partial x}\right]_j^n$,于是常用的4阶Runge-Kutta[1]方法为

$$\left.\begin{aligned}u^1&=u^n+\frac{1}{4}\Delta tR(u^n)\\u^2&=u^n+\frac{1}{3}\Delta tR(u^1)\\u^3&=u^n+\frac{1}{2}\Delta tR(u^2)\\u^4&=u^n+\Delta tR(u^3)\\u^{n+1}&=u^4\end{aligned}\right\} \tag{4}$$

实际上,我们的目标是增大时间步长,以减少趋于定常解的时间步,但式(4)增大由u^n到u^{n+1}的时间步长是靠多算了几步中间步后实现的,实际上它没有减少时间步,它所以受到人们的青睐在于增加中间步的同时,提高了时间方向的精度。因此,Runge-Kutta方法实际上靠提高精度实现了加速收敛。

取式(1)的隐式离散

$$\left(\frac{\partial u}{\partial t}\right)_j^{n+1}+a\left(\frac{\partial u}{\partial x}\right)_j^{n+1}=0,\ a>0 \tag{5}$$

对式(5)中的差分取一阶迎风差分得到

$$u_j^{n+1}=u_j^n-a\lambda(u_j^{n+1}-u_{j-1}^{n+1}) \tag{6}$$

式(6)和式(3)的精度相同,但式(6)无条件稳定。因此在增大时间步长,减少时间步方面最有效,最直接的方法是采取隐式离散,但是和显式离散相比,隐式方法的缺点在于:(Ⅰ)边界条件难于处理;(Ⅱ)向量运算时,难于求解;(Ⅲ)不象显式方法,易于结合Rung-Kutta方法提高时间方向的精度。

如何兼顾显、隐格式的优点,构造一个易于加速收敛的方法呢?作者针对模型方程作了详细的讨论。

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2　隐式加速收敛方法

为了兼顾显、隐格式的优点，即克服引言中提出的隐式格式的三个缺点，我们利用 Taylor 展开法，结合 TVD 格式，构造出一类具有显式边界，适合于向量运算的隐式加速收敛方法。方法反映一维粘性特性的模型方程为
$$\frac{\partial u}{\partial t}=-a\frac{\partial u}{\partial x}+v\frac{\partial^2 u}{\partial x^2},\quad a>0 \tag{7}$$
其中 a 为传播速度，v 为粘性系数。

将式(7)在时间方向作 Taylor 展开有
$$u_j^{n+1}=u_j^n+\Delta t\left(\frac{\partial u}{\partial t}\right)_j^n+\frac{\Delta t^2}{2}\left(\frac{\partial^2 u}{\partial t^2}\right)_j^n+\frac{\Delta t^3}{6}\left(\frac{\partial^3 u}{\partial t^3}\right)_j^n+O(\Delta t^4) \tag{8}$$
将 $\left(\frac{\partial^2 u}{\partial t^2}\right)_j^n=\frac{1}{2\Delta t}\left[\left(\frac{\partial u}{\partial t}\right)_j^{n+1}-\left(\frac{\partial u}{\partial t}\right)_j^{n-1}\right]+O(\Delta t^2)$

与 $\left(\frac{\partial^3 u}{\partial t^3}\right)_j^n=\frac{1}{\Delta t^2}\left[\left(\frac{\partial u}{\partial t}\right)_j^{n+1}-2\left(\frac{\partial u}{\partial t}\right)_j^n+\left(\frac{\partial u}{\partial t}\right)_j^{n-1}\right]+O(\Delta t^2)$

代入式(8)整理后得
$$u_j^{n+1}=u_j^n+\frac{5}{12}\Delta t\left(\frac{\partial u}{\partial t}\right)_j^{n+1}+\frac{8}{12}\Delta t\left(\frac{\partial u}{\partial t}\right)_j^n-\frac{1}{12}\Delta t\left(\frac{\partial u}{\partial t}\right)_j^{n-1}+O(\Delta t^4) \tag{8'}$$
令 $\delta u_j^{n+1}=u_j^{n+1}-u_j^n$，$\Delta u_j^n=\left(-a\frac{\partial u}{\partial x}+v\frac{\partial^2 u}{\partial x^2}\right)_j^n$

则式(8′)为
$$\delta u_j^{n+1}=\frac{5}{12}\Delta t\Delta u_j^{n+1}+\frac{8}{12}\Delta t\Delta u_j^n-\frac{1}{12}\Delta t\Delta u_j^{n-1}+O(\Delta t^4) \tag{9}$$
若式(8)中的 $\frac{\partial^2 u}{\partial t^2}$、$\frac{\partial^3 u}{\partial t^3}$ 取不同的差分逼近，则得到与式(9)不同的格式：
$$\delta u_j^{n+1}=\frac{4}{6}\Delta t\Delta u_j^{n+1}+\frac{1}{6}\Delta t\Delta u_j^n+\frac{1}{6}\Delta t\Delta u_j^{n-1}+O(\Delta t^3) \tag{10}$$
$$\delta u_j^{n+1}=\frac{3}{6}\Delta t\Delta u_j^{n+1}+\frac{4}{6}\Delta t\Delta u_j^n-\frac{2}{6}\Delta t\Delta u_j^{n-1}+\frac{1}{6}\Delta t\Delta u_j^{n-2}+O(\Delta t^3) \tag{11}$$
$$\delta u_j^{n+1}=\frac{3}{12}\Delta t\Delta u_j^{n+1}+\frac{14}{12}\Delta t\Delta u_j^n-\frac{7}{12}\Delta t\Delta u_j^{n-1}+\frac{2}{12}\Delta t\Delta u_j^{n-2}+O(\Delta t^4) \tag{12}$$
$$\delta u_j^{n+1}=\frac{10}{6}\Delta t\Delta u_j^n-\frac{5}{6}\Delta t\Delta u_j^{n-1}+\frac{1}{6}\Delta t\Delta u_j^{n-2}+O(\Delta t^3) \tag{13}$$

式(9)～式(13)都是关于时间 t 的半离散多步方法，前四式为隐式，最后一式为显式。实际应用时，考虑到稳定性和存储量一般选用式(9)、式(10)。到目前为止，我们还没有考虑空间物理残差 Δu_j^n 的离散形式，而 Δu_j^n 的离散形式对一个格式的稳定性、收敛性及捕捉激波的能力是至关重要的，由于 TVD 类格式是 l_1 收缩的[2]，且具有迎风特性，对激波的分辨率高，所以对 Δu_j^n 等我们选择三阶 TVD 离散[3]。但 TVD 类格式一般都比较复杂，对隐式方法来说，求解时很困难，尤其是推广到三维的时候，这一矛盾将十分突出，针对这一点，对 Δu_j^{n+1} 作适当的化简，将其转化为 δu_j^{n+1} 和 Δu_j^n 等，以降低求解难度。以式(9)为例
$$\delta u_j^{n+1}=\frac{5}{12}\Delta t(\Delta u_j^{n+1}-\Delta u_j^{n-1})+\frac{2}{3}\Delta t\Delta u_j^n+\frac{1}{3}\Delta t\Delta u_j^{n-1}+O(\Delta t^4) \tag{9'}$$
而
$$\begin{aligned}
&\Delta u_j^{n+1}-\Delta u_j^{n-1}=\left(-a\frac{\partial t}{\partial x}+v\frac{\partial^2 u}{\partial x^2}\right)_j^{n+1}-\left(-a\frac{\partial u}{\partial x}+v\frac{\partial^2 u}{\partial x^2}\right)_j^{n-1}=\\
&2\Delta t\left[\frac{\partial}{\partial t}\left(-a\frac{\partial u}{\partial x}+v\frac{\partial^2 u}{\partial x^2}\right)\right]_j^n+O(\Delta t^3)=\\
&-2a\Delta t\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)\right]_j^n+2v\Delta t\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right)\right]_j^n+O(\Delta t^3)=\\
&-2a\Delta t\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right)\right]_j^n+2v\Delta t\left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial u}{\partial t}\right)\right]+O(\Delta t^3)=\\
&-2a\frac{\Delta t}{\Delta x}\left[\left(\frac{\partial u}{\partial t}\right)_j^n-\left(\frac{\partial u}{\partial t}\right)_{j-1}^n\right]+2v\frac{\Delta t}{\Delta x^2}\left[\left(\frac{\partial u}{\partial t}\right)_{j+1}^n-\right.\\
&\left.2\left(\frac{\partial u}{\partial t}\right)_j^n+\left(\frac{\partial u}{\partial t}\right)_{j-1}^n\right]+\max[O(\Delta t^3),O(\Delta x^2\Delta t^3)]
\end{aligned}$$
于是式(9′)变为
$$A\delta u_{j-1}^{n+1}+B\delta u_j^{n+1}+C\delta u_{j+1}^{n+1}=\frac{2}{3}\Delta t\Delta u_j^n+\frac{1}{3}\Delta t\Delta u_j^{n-1}+O(\Delta t^4) \tag{9''}$$
其中 $A=-\frac{5}{6}a\frac{\Delta t}{\Delta x}-\frac{5}{6}v\frac{\Delta t}{\Delta x^2}$

$B=1+\frac{5}{6}a\frac{\Delta t}{\Delta x}+\frac{5}{3}v\frac{\Delta t}{\Delta x^2}$

$C=\frac{5}{6}v\frac{\Delta t}{\Delta x^2}$

式(9″)右端是已知量 Δu_j^n 与 Δu_j^{n-1} 的线性组合，和 Runge-Kutta 方法相比，虽然多了一项 Δu_j^{n-1} 使存储量有所增加，但和式(4)相比，同为时间方向的 4

阶精度，但减掉了中间步的计算，因此，总体上要比 Runge-Kutta 方法经济。另外，式(9″) 待求的未知量为 δu_j^{n+1}，且构成线性三对角方程，求解难度大大低于(9)。以 δu_j^{n+1} 为未知量，可使边界处理显式化，这是这一方法的另一优点。

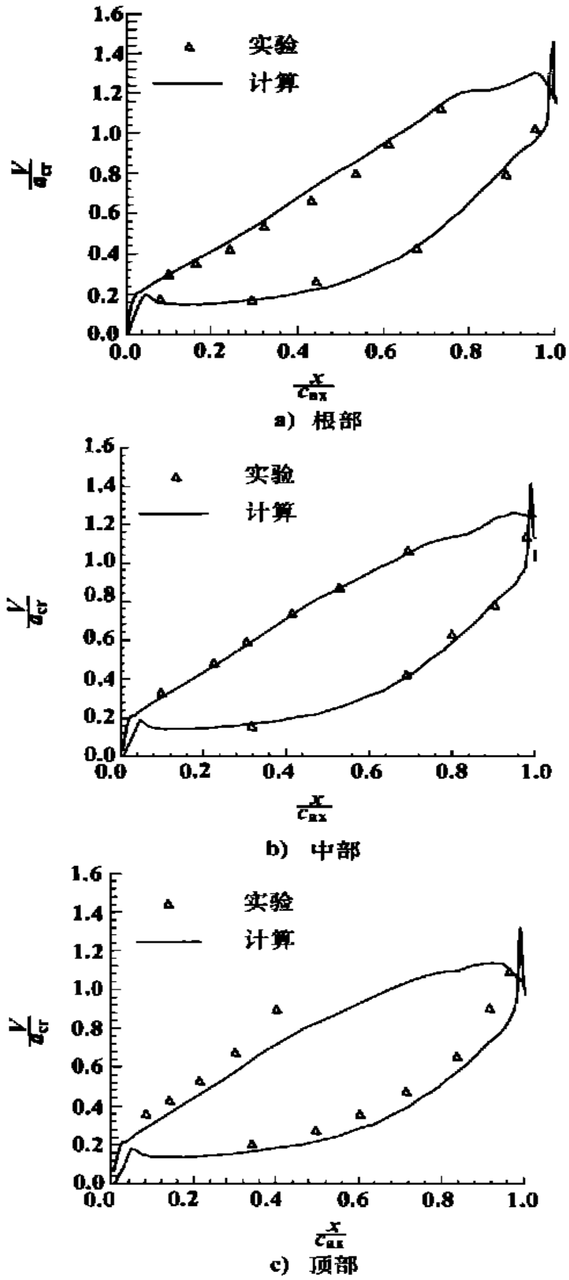

图1 叶型表面速度系数分布

从式(9) ~ 式(13)我们看到，由于都是从 Taylor 展开式等价转化来的，自动满足相容性。其表现形式就是，不管涉及到几层空间物理残差，其代数和都是一个时间步长 Δt。式(9″) 依然保持这一特性。另外这种半离散化处理，使空间物理残差的离散化可单独处理，格式灵活，使用方便。隐式项 δu_j^{n+1} 主要起增大时间步长的作用。显式项 Δu_j^n 等主要起提高激波分辨率的作用。

这种数值方法可推广到三维，只是时间方向的精度要降一阶，但是稳定性的时间步长依然可以取的很大，如下节的三维数值算例，CFL 数可达 10 以上。

3 数值算例

为了验证上述加速算法的可行性，用式(9)的三维推广式计算了 NASA Lewis 研究中心提供的高速涡轮试验，本次计算取 CFL 数为 10。计算给定进口总温 288.2 K、总压 1.013×10^5 Pa 及气流角为 0°。出口边界给定中径处的背压为 0.46×10^5 Pa，压力沿叶高分布由径向平衡方程给出。由于该试验仅提供了根部、中部及顶部三个截面的速度系数分布，因此图1只给出计算的根部、中部及顶部三个截面的叶型表面速度系数分布与实验结果的对比，由图中可以看出计算与试验结果吻合较好。

4 结论

(1) 作者在兼顾 CFL 数和精度的情况下，提出了一类具有显式边界，适合于向量运算的隐式加速收敛方法。该方法可推广到三维；

(2) 数值算例表明，本方法可靠易行，加速效果明显，由于 CFL 数可取到两位数(如本文数值算例 CFL 数取 10)，计算效率和一般的显格式相比，有了大幅度的提高；

(3) 对模型方程而言，其隐式项 Δu_j^{n+1} 的处理是在 $O(\Delta t^3)$ 的精度的条件下等价转化为 δu_j^{n+1} 与 Δu_j^n，和一般的隐格式相比，求解难度大大降低，而求解精度和求解效率有了很大的提高。

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A calculation model of heat and mass balance along with relevant software has been developed, which is fit for both flue gas desulfurization in a circulating fluidized bed (CFB-FGD) and various other kinds of semi-dry type flue gas desulfurization systems. The accurate calculation results of the above-mentioned model and software were verified with the help of a 1 t/h boiler flue gas desulfurization test rig and the operating data of several typical semi-dry type flue gas desulfurization systems currently in operation in China. The calculation model and software under discussion may well be employed for the design, operation and industrial control of semi-dry type flue gas desulfurization systems. **Key words**: semi-dry type flue gas desulfurization, spray drying, calcium-based absorbent, model

旋流煤粉多相流动与燃烧一维数学模型及应用＝**A One-dimensional Mathematical Model for Pulverized Coal Multi-phase Swirl Flow and Combustion and Its Applications** [刊, 汉] / CHEN Chun-ming, ZHANG Jian, ZHOU Li-xing (Department of Engineering Mechanics, Tsinghua University, Beijing, China, Post Code: 100084) //Journal of Engineering for Thermal Energy & Power. — 2001, 16(5). —533~536

With a view to developing an effective way of numerically simulating the multi-phase swirl flow of pulverized coal and its combustion set up was a one-dimensional mathematical model. Built on the framework of a multi-continuum model, the above-cited mathematical model comprehensively takes into account a gas-solid two-phase swirl flow as well as the pulverized coal combustion and heat transfer. It can be used to simulate in a speedy and effective way the pulverized coal multi-phase swirl flow and combustion process, as evidenced by the results of numerical calculation of pulverized coal and gas combustion in the annual duct of a vortex combustor. As a result, obtained were the following main parameters: in-furnace temperature, distribution of pulverized coal flow speed and concentration as well as combustion efficiency, etc. **Key words**: swirl multi-phase flow, pulverized coal combustion, one-dimensional mathematical model, vortex combustor

基于参数化建模的转子有限元剖分＝**Finite-element Meshing of a Turbine Rotor Based on Parametric Modeling** [刊, 汉] / WANG Zhang-qi, AN Li-qiang, PENG Zhen-zhong (Mechanical Engineering Department, North China Electric Power University, Baoding, Hebei Province, China, Post Code: 071003) //Journal of Engineering for Thermal Energy & Power. — 2001, 16(5). —537~539

The structural parametrization of a steam turbine rotor was attained through an analysis of its structural features. A method for the parametric modeling of the turbine rotor was put forward, resulting in an enhancement of the inputting efficiency and precision of the rotor initial geometric model. With the use of a Delaunay triangulation method generated by finite element grids a two-dimensional finite-element calculation model has been obtained. The finite-element grids feature a uniform size, a smooth transition in grid refinement and an absence of singular elements. As a result, fully ensured is the precision of finite element-based analytic calculation of the rotor temperature field and thermal stress. **Key words**: steam turbine rotor, parametrization, modeling, meshing

用时间相关法求解定常粘性流场的加速收敛法＝**Accelerating-convergence Approach for Solving a Steady Viscous Flow Field through the Use of a Time-marching Method** [刊, 汉] / ZHANG Yan-ying, WU Meng, SU Jie-xian (Energy Science and Engineering Institute under the Harbin Institute of Technology, Harbin, China, Post Code: 150001), CUI Ming-gen, Department of Mathematics, Harbin Institute of Technology, Harbin, China, Post Code: 150001) //Journal of Engineering for Thermal Energy & Power. — 2001, 16(5). —540~542

After a detailed analysis of the CFL number of explicit and implicit scheme it is noted that the key factor influencing the magnitude of CFL lies in a discrete form. To enhance the speed of convergence in solving a steady flow field when a time-marching method is used, it is essential to enlarge the CFL number. Meanwhile, the most direct and effective approach

for enlarging CFL number is to select an implicit scheme. Under the condition of meeting both a required CFL number and precision a combination of Taylor expansion method and TVD scheme can produce a kind of implicit method featuring an accelerating convergence. **Key words**: TVD scheme, implicit method, accelerating convergence, viscous flow field

船用主汽轮齿轮机组调节控制保安系统故障分析＝**Fault Analysis of a Regulation, Control and Security System for a Naval Main Steam Turbine-geared Unit** [刊,汉]/ CHEN Lin-gen (Power Engineering Institute under the Naval Engineering University, Wuhan, China, Post Code: 430033), LIU Wei-guo (Naval Representative Office at Dalian Shipyard, Dalian, China, Post Code: 116024)//Journal of Engineering for Thermal Energy &Power. —2001, 16(5). —543～545

Discussed are the failure forms and features of a regulation, control and security system for a new type of naval main steam turbine-geared unit. In addition, with an analysis of the underlying causes of faults and failures some countermeasures for their prevention are also proposed. **Key words**: main steam turbine-geared unit, regulation system, security system, fault analysis

一种锅炉燃烧控制的混合智能控制器＝**A Hybrid Intelligent Controller for a Boiler Combustion Control System** [刊,汉]/ LIANG Jian-wu, CHEN Yu-lin, ZHOU Cheng (Changsha Railway Engineering University, Changsha, Hunan Province, China, Post Code: 410075)//Journal of Engineering for Thermal Energy &Power. —2001, 16(5). —546～548

The combustion control system of a boiler is known to have a multitude of specific features, such as a strongly coupled state, being subject to a variety of outside disturbances, typical non-linearity, etc. In light of the above a hybrid intelligent control system based on "theory plus experience" is proposed, which combines PID (proportional-integral-differential) control, feed-forward control and expert control. As a result, the simplicity, reliability, anti-disturbance, rapid reaction and flexibility of the above three control methods are organically grouped into an integrated whole, giving full play to their respective merits. Initiating a new approach for industrial control technology the system under discussion has been successfully employed for the combustion control system of a 20 t/h boiler installed at Changsha Shuguang Electronic Tube Factory. **Key words**: PID control, feed-forward control, expert control, intelligent control

300 MW 火电机组仿真机给水调节方案的改进＝**An Improvement on the Feedwater Regulation Scheme for a 300 MW Thermal Power Plant Simulation Unit** [刊,汉]/ WANG Xiang-wei (Anhui Electric Power Simulation Center, Hefei, Anhui Province, China, Post Code: 230051)//Journal of Engineering for Thermal Energy &Power. —2001, 16(5). —549～551

In the course of the adjustment test of a simulation unit the design defects of the original control scheme of a feedwater automatic control system were analyzed and on this basis an improved scheme was proposed. After a modification of the related design modules and procedures of the simulation unit the regulation quality of the feedwater regulation system has been greatly enhanced. This brought about the realization of a formerly unachievable protection function, which has been verified and confirmed in production practice. It is noted that the simulation unit can be employed not only to conduct on-the-post training but also possesses a highly effective adjustment-test function. **Key words**: simulation, feedwater automatic control, adjustment test, multi-circuit balanced output module

一台余热锅炉受热面积灰和烟气除尘的改进分析＝**An Analysis of the Measures for Alleviation of Ash Buildup on the Heating Surfaces of a Heat Recovery Boiler and the Enhancement of Dust and Smoke Removal Efficiency** [刊,汉]/ HAN Jia-de, LU Yi-ping (Mechanical Engineering Institute under the Harbin University of Science and Technology, Harbin, China, Post Code: 150080)//Journal of Engineering for Thermal Energy &Power. —2001, 16(5). —552～553