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具有辐射边界的三维非规则域内稳态温度场分析

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摘　要: 研究了具有辐射边界的空间非规则域内稳态导热问题, 求解方法为在球极坐标系内分离变量, 获得级数形式的解后, 采用边界离散法确定级数项的待定系数。算例表明, 边界离形方法不仅可以解决非正交边界问题, 而且也可以处理诸如辐射边界的非线性边值问题。

关　键　词: 温度场; 边界离散; 非正交边界; 非线性边值

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1　引言

非规则域是解析法解决数学物理问题中较为棘手的问题。对于一些简单的非规则域, 如果是平面问题, 可以通过保角映射, 使其化为在特定坐标系具有正交边界的计算域; 如果是空间问题, 则保角映射也无能为力, 这会给解析法带来许多困难。虽然非正交边界与正交边界一样, 也可以使用经典的 Fourier-Liouvill 积分法。但对于非正交边界, Fourier-Liouvill 积分法的计算是非常复杂的, 不仅需要进行大量的 Fourier-Liouvill 积分, 而且还需要求解未知量彼此耦联的无穷代数方程组, 不便于实际应用。此外, 建立在线性问题基础上的解析法在处理非线性边界条件时更为困难, 一般难于直接求解, 只能通过分段线性化处理, 或采用数值方法求解。可见, 具有辐射非线性边界条件的空间非规则域内的导热问题, 解析求解是相当困难的。

边界离散方法[1] 是一种半解析方法, 就其本质而言, 它也是一类边界元法, 与通常的边界元法不同之处在于, 其基本解不是建立在 Green 函数基础上的单层或多层势组成的积分结构, 而是以分离变量所构造的特殊函数作为基本解, 将问题的解表达为显式的无穷级数, 在分离变量形式解的基础上将边界离散为有限多个离散点, 然后在每个离散点上, 根据边界条件对形式解直接赋值, 建立以形式解级数项待定系数为未知量的代数方程组, 如果取边界离散点的个数恰好等于形式解被截断后级数项待定系数的个数, 则求解上述代数方程组就可以确定这些待定系数, 从而得到问题解析形式的解。应当指出, 边界上离散点(即拟合点)的选择有一定任意性, 得到解答后可以选择一组非拟合的边界点来检验边界条件的吻合程度。如果要进一步改善精度, 则可根据拟合误差的最小二乘原则, 通过单纯形法[2] 局部寻优, 选出质量更好的离散点位。但一般情况下, 若离散点具有足够密度, 寻优步骤可以省略。

三维空间的非线性边值问题以及相应的反问题一般都涉及巨量的运算, 因此不断改进运算策略和算法是必要的。本文将以具有辐射边界的三维非规则域的导热问题为例来阐明所论算法的有效性。

2　物理模型及定解问题

设有处于真空中的空心球体, 空心部分的形状为椭球体。截面形状如图 1 所示, 计算域记为 Ω, 内、外边界面分别记为 S_i 和 S_o。金属导热系数为 λ(W/m℃), 内壁维持在温度 T_0(℃), 裸露的外壁是具有发射率为 ε 的漫射 — 灰体表面, 它将能量辐射到 $T_e \approx 0$ 的周围环境中[3~4]。该问题属于非正交边界域内具有非线性边界条件的稳态热传导问题, 其定解问题可描述为

控制方程

$$\nabla^2 T = T_{rr} + \frac{1}{r}T_r + \frac{1}{r^2}T_{\theta\theta} + \frac{\operatorname{ctg}\theta}{r^2}T_\theta + \frac{1}{r^2\sin^2\theta}T_{\Phi\Phi} = 0 \tag{1}$$

$$(r \in \Omega, 0 \leqslant \theta \leqslant \pi, 0 \leqslant \Phi \leqslant 2\pi)$$

式中 T 为温度; (r, θ, Φ) 为球极坐标。

边界条件

$$T = T_i \quad (\text{在 } S_i \text{ 上}) \tag{2}$$

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$$\frac{\partial T}{\partial n}+\frac{\sigma\varepsilon}{\lambda}T^4=0 \quad (在\ S_o\ 上) \qquad (3)$$

式中 σ 为 Stefan-Boltzmann 常数; n 为 S_o 的外法线方向的距离变量。

式(3) 中, 辐射边界条件是非线性的, 它与边界温度的四次方有关, 而边界温度也是待求的。对此非线性边界条件的处理, 通常的分析法是将其线性化[3, 5], 例如使温度的幂指数由四次降为一次。采用本文的边界离散法处理这类问题时, 无须对式(3) 作线性化处理, 而保持其非线性特征, 并且可以得到解析形式的解。与线性边界条件问题所不同的是, 对于非线性边界条件, 需要求解非线性代数方程组, 而不是线性代数方程组。

3　边界离散和方程求解

采用分离变量法, 设偏微分方程解的形式为

$$T(r,\theta,\varphi)=R(r)\Theta(\theta)\Phi(\Phi) \qquad (4)$$

解得

$$\Phi(\Phi)=A\cos m\Phi+B\sin m\Phi$$

$$R(r)=Cr^n+Dr^{-n}$$

$$\Theta(\theta)=EP_n^m(\cos\theta)$$

式中 A、B、C、D、E 均为系数; $P_n^m(\cos\theta)$ 为 n 阶伴随 Legendre 函数。

式(1) 的通解为[6]

$$T(r,\theta,\varphi)=\sum_{n=0}^{\infty}\sum_{m=0}^{n}(a_{nm}r^n+b_{nm}r^{-n-1})P_n^m(\cos\theta)\times\cos m\Phi+\sum_{n=1}^{\infty}\sum_{m=1}^{n}(c_{nm}r^n+d_{nm}r^{-n-1})P_n^m(\cos\theta)\sin m\Phi \qquad (5)$$

如果将形式解(5) 的无穷级数截断为有限项 N, 则形式解中共有 $2(N+1)^2$ 个待定的级数项系数 $(a_{nm}, b_{nm}, c_{nm}, d_{nm})$。

4　边界离散法求待定系数

按照边界离散法, 在边界面 S_i 和 S_o 上分别取 $(N+1)^2$ 个离散点(均布), 设点 P_i 和 P_j 分别为边界面 S_i 和 S_o 上某一个离散点, 它们的坐标分别为 $(r_b^i, \theta^i, \Phi^i)$ 和 $(r_b^j, \theta^j, \Phi^j)$, 在每一个边界离散点上将边界条件式(2) 和式(3) 代入形式解(5), 得

$$\sum_{n=0}^{\infty}\sum_{m=0}^{n}[a_{nm}(r_b^i)^n+b_{nm}(r_b^i)^{-n-1}]P_n^m(\cos\theta^i)\times\cos m\Phi^i+\sum_{n=1}^{\infty}\sum_{m=1}^{n}[c_{nm}(r_b^i)^n+d_{nm}(r_b^i)^{-n-1}]\times P_n^m(\cos\theta^i)\sin m\Phi^i=T_0 \qquad (6)$$

$$[i=1,2,\cdots,(N+1)^2]$$

$$\sum_{n=0}^{\infty}\sum_{m=0}^{n}[na_{nm}(r_b^j)^{n-1}-(n+1)b_{nm}(r_b^j)^{-n-2}]\times P_n^m(\cos\theta^j)\cos m\Phi^j+\sum_{n=1}^{\infty}\sum_{m=1}^{n}[nc_{nm}(r_b^j)^{n-1}-(n+1)d_{nm}(r_b^j)^{-n-2}]P_n^m(\cos\theta^j)\sin m\Phi^j+\Big\{\sum_{n=0}^{\infty}\sum_{m=0}^{n}[a_{nm}(r_b^j)^n+b_{nm}(r_b^j)^{-n-1}]P_n^m(\cos\theta^j)\cos m\Phi^j+\sum_{n=1}^{\infty}\sum_{m=1}^{n}[c_{nm}(r_b^j)^n+d_{nm}(r_b^j)^{-n-1}]P_n^m(\cos\theta^j)\sin m\Phi^j\Big\}^4=0 \qquad (7)$$

$$[j=1,2,\cdots,(N+1)^2]$$

联立式(6) 和式(7), 得到关于 $(a_{nm}, b_{nm}, c_{nm}, d_{nm})$ 的由 $2(N+1)^2$ 个方程组成的非线性代数方程组。采用拟牛顿法[1] 求解。得到收敛解后即为所求形式解的级数项待定系数, 然后将其代回到式(5), 最终得到级数形式的解。

5　算例分析

对图 1 所示具有辐射边界的空心球体的温度场进行分析。取计算域材质分别为不锈钢 $\lambda=12$ W/(m·℃)、纯铝 $\lambda=204$ W/(m·℃)、纯银 $\lambda=419$ W/(m·℃), 球体外半径 $R=0.5$ m, 内部空心椭球的三个半轴分别为 $a=b=0.4$ m, $c=0.3$ m, $\varepsilon=0.65$, $T_0=373$ K。计算结果如表 1、表 2 所示(取 $N=2$)。表 1 为 Ⅰ－Ⅰ 截面径向温度分布, 表 2 为 Ⅱ－Ⅱ 截面径向温度分布。可以看出, 材料导热系数大时, 球体外表面温度高; 材料导热系数一定时, 薄壁处球外表面温度高, 这与实际情况都是吻合的。

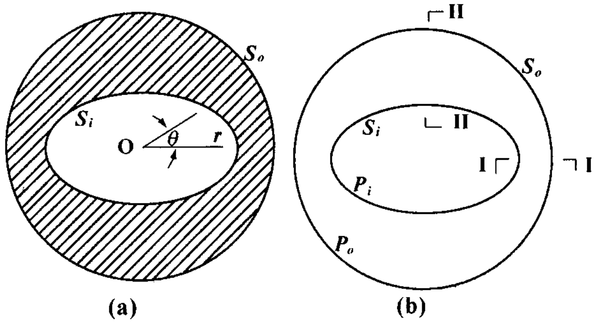

图 1　计算域剖面及边界离散

表 1　Ⅰ－Ⅰ 截面径向温度分布

r/m	不锈钢	纯铝	纯银
0.40	373.000	373.000	373.000
0.42	371.070	372.876	372.939
0.44	369.381	372.767	372.886
0.46	367.895	372.672	372.840
0.48	366.582	372.588	372.799
0.50	365.419	372.514	372.763

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成金属 Fe 后才能使致密的固态 Fe 继续裂开，不断地形成新的界面。因此，Fe、NO、CO 反应界面是由 Fe、NO 和铁氧化物 CO 两系统中反应速率较慢的一个确定。根据化学反应计量关系，可以得到如下反应时间 t 与界面位置 r 的关系：

$$\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{bk_{ci}C_i}{a\rho_{si}} \tag{7}$$

其中：k_{ci} — 表观反应速率常数，C — 反应界面处反应物浓度，可由 Langmuir 吸附等温式确定，ρ_s — 固体密度，i — 分别代表 NO 和 CO。

初始条件：$t=0$，$r=R_0$。所以：

$$r=\min\left(R_0-\frac{bk_{ci}C_i}{a\rho_{si}}t\right) \tag{8}$$

中间产物微粒的反应情况：由于处于固态 Fe 不同径向位置微粒的生成时刻各不相同，它们的反应程度也不一致。假定已经进行的总反应时间为 t，对于介于 r、R_0 之间任意位置 R 处的微粒，其参加反应的时间 t_w 为：

$$t_w=t-t(R) \tag{9}$$

式中：$t(R)$ — 裂孔反应界面到达 R 处所需要的时间。

于是，可以得到中间产物微粒的反应界面 r_w 的进展：

$$r_w=\min\left(R_{0w}-\frac{bk_{ci}C_{i,w}}{a\rho_{si}}t_w\right) \tag{10}$$

式中：$C_{i,w}$ — 微粒反应界面 NO、CO 的浓度。

至此，可以确定总的 Fe 转换率。上述模型只是对 Fe、NO、CO 复杂反应的初步分析，其中的许多参数，如气体在裂孔过程中的扩散、微粒尺寸和孔隙率等的统计特性还有待于进一步的实验和理论分析。

5　结论

在 CO 作用下 Fe 可以高效地催化还原 NO。在静态实验条件下，1 123 K 时，在 CO 作用下，NO 的转化率为 70%。Fe、NO 和 CO 系统中，反应后的表面呈现出多孔的特点，促使 NO 浓度的持续降低。分析认为，在高温条件下 NO 在铁氧化物上的吸附能力比 CO 强，Fe、NO、CO 反应过程中反应界面由 Fe、NO 或铁氧化物、CO 中反应速率慢的确定。

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表 2　II — II 截面径向温度分布

R/m	不锈钢	纯铝	纯银
0.30	373.000	373.000	373.000
0.32	371.498	372.906	372.954
0.34	369.664	372.781	372.893
0.36	367.962	372.665	372.836
0.38	366.382	372.557	372.783
0.40	364.915	372.458	372.735
0.42	363.548	372.365	372.689
0.44	362.274	372.278	372.647
0.46	361.083	372.197	372.607
0.48	359.967	372.122	372.570
0.50	358.918	372.050	372.535

在以上算例中，对边界非拟合点的检验均符合精度要求，不需进行离散点局部寻优程序。

6　结论

对于空间非规则域与非线性辐射条件组合的稳态导热问题，边界离散发的求解是非常成功的。所以，边界离散法不仅适用于大量可分离变量的非正交边界问题，而且适用于非线性边值问题。

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line training were conducted simultaneously. The neural network training by the use of the Powell method features a rapid and stable training process. It has the merits of the ability to perform on-line learning and diagnose the failure of a multiple of sensors. Actual tests on boilers show that the above method is highly effective. **Key words**: neural network, failure detection, failure diagnosis, sensor

燃气动力装置性能参数的热经济性分析与决策＝**Thermodynamic-effectiveness Analysis and Decision-making for the Performance Parameters of a Gas Power Plant** [刊,汉] / LI Shi-wu, (Department of Aeronautical Power and Thermal Energy Engineering, Northwestern Polytechnic University, Xi'an, China, Post Code: 710071) //Journal of Engineering for Thermal Energy & Power. —2002, 17(1). —76～79, 83

Thermodynamic effectiveness features the energy utilization economy of an equipment item under the condition of its having attained a given technical objective. Through an analysis of the performance parameters of a gas power plant from the perspective of enhancing economy it has been found that thermal efficiency is not fit to serve as the decision-making index of thermodynamic effectiveness for the selection of performance parameters. By contrast it is more rational to designate the plant operating cost as a decision-making index, because it has taken into account both the design and operation factors. With a constant-pressure heating cycle-based gas turbine power plant serving as an example a thermodynamic-effectiveness optimization model has been set up along with the determination of its thermodynamic-effectiveness performance parameters. The above example can be used to prove that the seeking and use of thermodynamic-effectiveness performance parameters may be considered as a new method for the design decision-making of a gas power plant. **Key words**: gas power plant, design, thermodynamic effectiveness, decision-making

汽机调节阀阀体三维瞬态温度场及应力场分析＝**Three-dimensional Transient Temperature Field of the Valve Body of a Turbine Regulating Valve and the Analysis of Its Stress Field** [刊,汉] / PENG Zhen-zhong, DING Zhu-shun, WANG Zhang-qi, WANG Song-ling (North China Electric Power University, Baoding, Hebei Province, China, Post Code: 071003) //Journal of Engineering for Thermal Energy & Power. —2002, 17(1). —80～83

An effective method is proposed for the modeling of a valve body with the help of a finite element method. Through the use of a structural-analysis finite element method an analytical calculation was conducted of the valve body of a main steam regulating valve for a Chinese-made 125 MW steam turbine. It includes such a variety of items as the valve body temperature field, thermal stress field, mechanical stress field and comprehensive stress field under the startup and shut-down operating conditions respectively at cold, warm and hot states. As a result, obtained were the detailed temperature field at key points under cold startup and shut-down operating conditions as well as the variation relationship of its corresponding thermal stress fields. In addition, also presented are the stress field calculation results of the valve body under the warm and hot startup and shutdown operating conditions with the loss of valve body service life being evaluated at various-state startups. **Key words**: valve body, finite element, temperature field, stress field

具有辐射边界的三维非规则域内稳态温度场分析＝**An Analysis of the Steady-state Temperature Field in a Spatial Irregular Domain with a Radiation Boundary** [刊,汉] / LIU You-jun (Beijing Polytechnic University, Beijing, China, Post Code: 100000), FAN Hong-ming (Tsinghua University, Beijing, China, Post Code: 100084), HE Zhong-yi (Harbin Institute of Technology, Harbin, China, Post Code: 150009) //Journal of Engineering for Thermal Energy & Power. —2002, 17(1). —84～85, 89

Studied is a steady-state heat conduction problem in a spatial irregular domain with a radiation boundary. The following method has been employed to solve the problem. With variables being separated in spatial spherical coordinates obtained is a solution of series mode. Then, by using a boundary discrete method one can obtain the factor of the series term to be determined. The calculation example indicates that the boundary discrete method can be used to solve not only non-orthogonality boundary problems, but also problems of nonlinear boundary (such as a radiation boundary) value. **Key words**: temperature field, boundary discrete method, non-orthogonality boundary, nonlinear boundary value

一氧化碳作用下铁对一氧化氮的催化还原实验与动力学过程分析＝**Experimental and Kinetics Process Analysis of NO Catalytic Reduction by Iron under the Action of CO** [刊, 汉] / ZHOU Hao-sheng, LU Ji-dong, ZHOU Hu, et al (National Key Lab of Coal Combustion under the Huazhong University of Science and Technology Wuhan, China, Post Code: 430074) // Journal of Engineering for Thermal Energy & Power. —2002, 17(1). —86～89

An experiment and analysis was conducted of the catalytic reduction process of NO by iron under the action of CO. It has been found that at a temperature of 1123 K the conversion rate of NO to N_2 was 70%. A very porous structure resulted after a reaction of NO with Fe and CO. An analysis indicates that under high temperatures the absorption ability of NO on iron oxides is stronger than that of CO, resulting in the presence of iron oxides on the reaction surface. It is assumed that the reaction interface in the reaction process was decided by the slower reaction rate between Fe and NO or between iron oxides and CO. On this basis set up preliminarily was a physical and mathematical model for the above reaction. **Key words**: nitric oxide, iron, catalytic reaction, kinetics

中小型煤粉炉的运行优化＝**Optimized Operation of Medium and Small-sized Pulverized Coal-fired Boilers** [刊, 汉] / LU Ze-hua, XU Chun-hui (Thermal Energy Engineering Department, Tsinghua University, Beijing, China, Post Code: 100084) // Journal of Engineering for Thermal Energy & Power. —2002, 17(1). —90～92

The enhancement of operating efficiency of medium and small-sized pulverized coal-fired boilers with their operation controlled at an optimum air-coal ratio has always been a difficult issue. This comes about because the optimum air-coal ratio varies with boiler conditions, boiler load and ranks of coal fired and assumes a non-steady magnitude. Proceeding from the reverse balance method of boiler efficiency calculation and by taking furnace outlet temperature and exhaust gas temperature as major factors the authors have derived by logical reasoning the thermal efficiency judgement criteria for seeking an optimum air-coal ratio. On this basis a knowledge base was set up by a self-study system. As a result, a two-dimensional fuzzy decision table can be obtained. It has the boiler load and coal rank serving as parametric variables and the oxygen content of flue gas, which characterizes the optimum air-coal ratio, serving as dependent variables. This approach has solved the difficult problem of how to attain the high-efficiency operation of medium and small-sized pulverized coal-fired boilers during their on-line and real-time control. **Key words**: optimum air-coal ratio, thermal efficiency judgement criteria, fuzzy decision table, optimized operation

温度场分析的自适应有限元方法＝**Self-adaptive Finite Element Method for the Analysis of a Temperature Field** [刊, 汉] / WANG Zhang-qi, AN Li-qiang (Mechanical Engineering Department, North China Electric Power University, Baoding, Hebei Province, China, Post Code: 071003) // Journal of Engineering for Thermal Energy & Power. —2002, 17(1). —92～94

Self-adaptive finite element method has its important application value in the analysis of practical engineering problems. In the light of the specific features of a temperature field the authors have presented a simple method for the estimation of local errors in the finite element calculation of a temperature field. In conjunction with Delaunay triangulation method of finite element mesh generation a study was conducted of the self-adaptive method for the finite element analysis of a temperature field. In addition, the error estimation of the temperature field and other functions, such as automatic division of