文章编号: 1001－2060(2002)01－0092－03

温度场分析的自适应有限元方法

王璋奇, 安利强

(华北电力大学 机械工程系, 河北 保定　071003)

摘　要: 自适应有限元方法在实际工程问题的分析中有重要的应用价值。本文根据温度场分析的特点, 给出了温度场有限元计算结果局部误差估计的简单方法, 结合有限元网格生成的 Delaunay 三角化方法, 研究温度场有限元分析的自适应方法, 并在汽轮机转子温度场有限元分析软件中成功实现了温度场的误差估计和有限元网格的自适应自动剖分、加密等功能, 获得了具有等精度的温度场有限元数值计算结果。以国产 300 MW 汽轮机转子温度场分析的有限元网格和计算结果为例, 说明了本文方法的有效性。

关　键　词: 有限元; 网格剖分; 自适应; 温度场

中图分类号: TB115　　文献标识码: A

1　引言

随着计算机技术及数值计算方法的发展, 有限元技术的应用领域不断扩大, 许多大型复杂的结构中物理场分析都采用有限元法。有限元计算中网格的精度直接关系到计算结果的精度, 在有限元实际计算过程中, 往往要求在保证计算精度的前提下, 采用最少的单元, 以减小计算的工作量。但是在计算完成前, 一般不能准确的判断出所分析的物理场的分布情况, 因而无法决定网格的疏密分布, 只能凭经验在梯度可能较大的部位进行网格加密, 这样势必使得网格加密带有一定程度的盲目性, 造成资源的浪费。

工程上要求了解数值计算的具体误差值, 以便判断近似解的可靠程度, 客观上需要求解过程以及解的特征信息, 以确定计算结果的误差分布状况。这种误差估计与有限元网格生成和改进有机地结合起来, 形成一个渐进地分析过程, 使合乎精度要求的有限元解和网格同步产生, 即自适应有限元过程[1]。

采用自适应的有限元分析技术, 可以由分析软件根据计算结果的精度, 自动决定网格的密度, 在初次剖分不满足精度要求的局部区域进行自动加密, 极大的提高了计算精度, 减少了有限元前处理的工作量, 使得有限元技术在工程技术中的应用更加广泛。

Delaunay 三角化方法适应于事后的网格局部加密。可通过在原有网格的基础上插入新节点, 然后进行 Delaunay 算法处理, 可以实现网格的自动加密[2]。本文以此网格剖分算法为基础, 提出了温度场有限元分析的自适应方法, 并在软件中实现。

2　基于 Delaunay 三角化的网格剖分

任意二维区域有限元网格剖分的 Delaunay 算法已经有许多的论述[1]。其主要实现步骤如下[3,6]。

(1) 根据给定各边界的权函数生成边界节点(包括内环和外环)。

(2) 将边界节点进行 Delaunay 处理, 生成开端网格。

(3) 根据边界节点的权函数, 逐步生成内部节点, 并进行 Delaunay 处理, 直至满足剖分要求。

按照此剖分方法, 在网格的生成过程中, 内部节

收稿日期: 2001－01－19; 修订日期: 2001－10－18

作者简介: 王璋奇(1964－), 男, 陕西大荔人, 华北电力大学教授, 博士.

由此模糊决策表可以得到如下结论: 当负荷增高和煤种变好时, 调整总风量应使烟气含氧量下降将使锅炉的效率提高。不同煤种、不同负荷时, 烟气含氧量的最佳值可方便地查表得到, 解决了如何使煤粉炉在线运行在最佳风煤比处以提高运行效率的难题。

参考文献:

[1] 冯俊凯, 沈幼庭. 锅炉原理与计算[M]. 北京: 科学出版社, 1992.

[2] 戴绪愚. 自寻最优控制[M]: 北京: 科学出版社, 1986.

[3] 徐春晖. 中小煤粉炉燃烧控制的主导因素法[D]. 北京: 清华大学, 1996.

(辉　编辑)

点的生成是由边界节点的权函数控制的, 这样就可以实现网格的均匀过渡, 得到满意的有限元网格。本文在有限元网格的自动生成和自动加密过程中都采用了 Delaunay 算法。

3　温度场局部误差估计方法

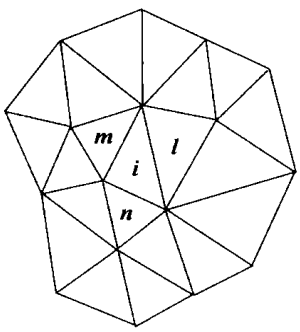


图 1　活跃单元

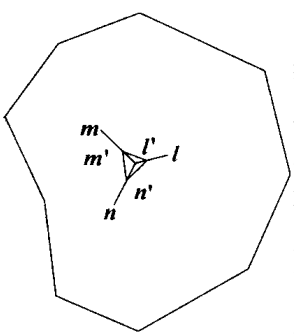


图 2　加密区域

实际工程问题有限元求解结果的误差计算是正确判断近似解的可靠程度的关键。根据实际需要, 人们已提出了多种误差估计方法, 并成功地应用于具体工程问题的数值计算结果的误差估计[4~5]。在这些计算误差估计的方法中, 应用了大量的有限元求解过程信息, 并需要对结果进行复杂的运算, 增加分析工作量[4~5]。在温度场数值计算的课题研究过程中, 温度场变化梯度大的地方, 其网格密度应该大一些; 温度场变化梯度小的地方, 其网格密度应该疏一些。这一直观的控制计算结果精度的准则, 其实质就是要求相邻单元内的温度变化不能过大。据此本文给出一种简单的温度场的局部误差计算方法, 定义单元温度场的局部误差就是该单元与其相邻的单元温度的最大差值和该单元的比值。按照此定义, 单元 i 的局部误差 δ 的计算公式如下:

$$\delta = \frac{\max(\Delta)}{T[i]} \times 100\%,\ j = 1, 2, 3, \cdots, m \qquad (1)$$

公式中, $\Delta = T[i] - T[j]$, $T[i]$ 为 i 单元的温度, $T[j]$ 为与 i 单元相邻单元的温度, 相邻单元总数为 m 个。

计算实践证明, 这一近似估计有限元温度场计算结果精度的方法是有效的, 且计算量小、容易实现。

4　网格局部加密的方法

在网格自适应加密的过程中, 根据对有限元计算结果的精度要求, 首先确定局部误差的极限值 ε。然后按单元搜索, 如果 i 单元的精度不满足要求, 即 $\delta > \varepsilon$。则该单元为局部加密的活跃单元。

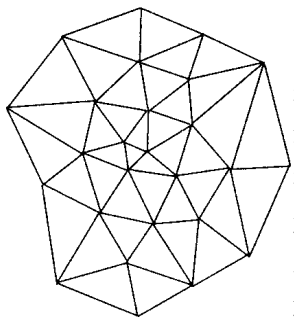


图 3　加密后网格

根据不满足精度单元的位置, 其网格的局部加密方法分为两种, 一种是活跃单元为内部单元的加密过程, 即该单元的所有边为内部边。第二种是活跃单元为边界单元的加密过程, 即单元的一条或两条边为边界。由于在网格生成过程中, 这两种单元的生成算法不同, 因而实现局部加密方法也不同。

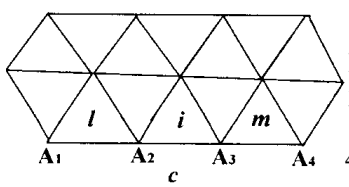


图 4　活跃单元

4.1　内部局部加密方法

设活跃单元为 i, 找到与该单元有公共边的 3 个单元, 记为 l, m, n, 如图 1 所示。分别求四个单元的形心, 记为 i, l, m, n, 做连线 il, im, in, 并在三条连线上进行线性插值, 分别求出温度值为 $\varepsilon T[i]$ 的点 l', m', n', 如图 2 所示。

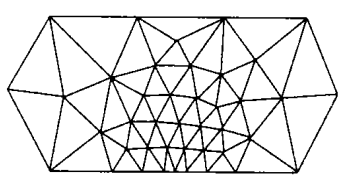


图 5　加密后网格

计算节点坐标的插值公式为:

$$\begin{cases} x'_j = x_i + \dfrac{\varepsilon}{\Delta}(x_j - x_i) \\ y'_j = y_i + \dfrac{\varepsilon}{\Delta}(y_j - y_i) \end{cases} \quad j = l, m, n \qquad (2)$$

删去 i, l, m, n 四个单元、节点及含被删除节点的相关单元。这样, 由 l', m', n' 三点组成一个单元 $l'm'n'$, 将其作为内环, 由删除单元后剩余的单元形成的边界作为外环, 组成一个二维多连域, 如图 2 所示。将其作为任意二维区域用上述的 Delaunay 三角化过程进行剖分, 以获得加密后的网格, 如图 3 所示。

采用此法进行局部加密, 内环 $l'm'n'$ 上的点的权函数(即三角形 $l'm'n'$ 的边长) 可以控制该局部区域的网格密度, 从而保证加密后, 该局部网格密度发生变化, 改善计算结果的精度, 使其达到精度要求。外环由原网格模型的节点组成, 不进行加密, 从而保证了局部加密后的网格与原网格匹配。

4.2 边界局部加密方法

在无内热源的温度场实际计算中, 边界往往是温度梯度较大的部位, 因而, 边界区域的精度一般不易满足要求, 需要对边界单元进行加密处理。边界单元的加密方法与内部单元完全不同, 下面介绍边界局部加密的方法。

假定边界单元 i 不满足精度要求(如图 4), 则删除此边界单元, 及其相邻的两个边界单元 l, m, 同时删除与 i, l, m 单元相关的内部单元。设 A_1, A_2, A_3, A_4 为相邻三个边界单元的边界点, 取单元 i 对应的边界 A_2A_3 边的中点为 C, 它将边界 A_1A_4 分成 A_1C, CA_4 两部分。这样由 A_1C, CA_4, 及删除单元的其余边界组成一个二维单连域, 如图 4 所示。重新确定边界 A_1C 和 CA_4 的权函数, A_1C 边界 A_1 端的权函数为 $\bar{A}_1\bar{A}_2$, C 端的权函数为 L_l, CA_4 边界 A_4 端的权函数为 $\bar{A}_3\bar{A}_4$, C 端的权函数为 L_m, 权函数 L 的计算公式为

$$L_j = \frac{\varepsilon}{\Delta_j} \mid ij \mid \quad j = l, m \tag{3}$$

式中 $\mid ij \mid$ 为 i 单元形心与 j 单元形心的距离。

根据边界的权函数, 首先生成边界 A_1C 和 CA_4 上的边界点, 然后按第一部分叙述的剖分方法进行剖分。

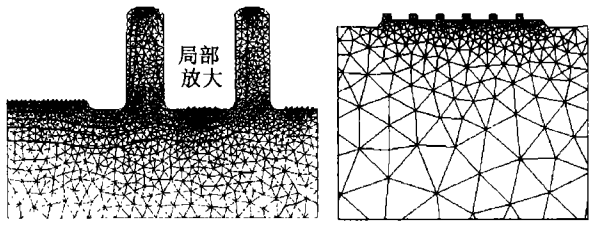
采用此法加密, 边界 A_1C 和 CA_4 的 C 端的边界权函数可以控制该局部区域的加密, 从而保证加密后该局部精度提高, 达到精度要求。A_1 和 A_4 端的权函数取为原权函数, 可以保证单元的均匀过渡, 其余边界节点由原网格的节点组成, 没有进行加密, 从而保证了局部加密后的网格与原网格匹配。

5 实例分析

在研制转子温度场有限元分析软件包的过程中, 笔者根据本文提出的网格加密方法, 结合 Delaunay 三角化, 完成了任意二维区域有限元网格的生成与加密。利用本文方法处理汽轮机转子温度场计算问题, 可方便、迅速地完成转子求解区域的有限元网格生成和加密, 简化了分析过程, 提高了分析精度。

利用该方法对国产 300 MW 汽轮机高压转子冷启动温度场进行了分析计算, 自动加密后的有限元网格以及转子外表面蒸汽加热过程中某时刻的等温线如图 6 所示。

由图中可见, 该处理过程所得到的有限元网格大小均匀, 几乎不会出现"狭长类"单元, 网格疏密过渡平缓, 保证了物理场量的连续性, 并在变化梯度大的区域网格较密, 实现了物理场量的等精度原则。



(a) 整体网格及等温线 (b) 网格局部放大

图 6 自动加密转子有限元网格及某时刻等温线

6 结论

(1) 本文提出的温度场有限元计算结果的误差估计方法是有效的, 以此为基础的温度场自适应有限元方法是正确的。利用本文的方法可以得到具有等精度的温度场有限元数值计算结果。

(2) 采用本文提出的自适应方法, 处理过程简单, 采用 Delaunay 算法, 得到的有限元网格均匀, 网格的疏密过渡均匀, 可避免影响有限元计算精度的奇异单元。

(3) 在汽轮机转子温度场计算课题的研究过程中, 应用本文提出的自适应有限元方法实现的汽轮机转子温度场自适应分析, 提高了计算结果精度和分析的效率。

参考文献:

[1] BABUSKA I, ZIENKIEWICZ O C. Accuracy estimates and adaptive refinements in finite element computations[M]. John Wiley and Sons, 1986.

[2] X XU, PAIN C C, GODDARD A J H. An automation adaptive meshing technique for delaunay triangulation[J]. **Computers Methods Applied mechanics and Engineering**, 1998, **16**(2): 297—303.

[3] WEATHERILL N. Delaunay triangulation in computational fluid dynamics[J]. **Comput Math Appl**, 1992, **24**(1): 129—150.

[4] ZIENKIEWICZ O, ZHU J. A simple error estimator and adaptive procedure for practical engineering analysis[J]. **Int J Numer Methods Engrg**, 1987, **24**(2): 337—357.

[5] BIDDLECOMBE C S, SIMKIN J, TROWBRIDGE C W. Error analysis in fine element models of electromagnetic fields[J]. **IEEE Trans. On MAG**, 1986, **22**(5): 811—813.

[6] 袁建生. 一种高性能二维有限元网格剖分软件[J], 华北电力学院学报, 1998, **15**(1): 38—55.

(辉 编辑)

is a solution of series mode. Then, by using a boundary discrete method one can obtain the factor of the series term to be determined. The calculation example indicates that the boundary discrete method can be used to solve not only non-orthogonality boundary problems, but also problems of nonlinear boundary (such as a radiation boundary) value. **Key words**: temperature field, boundary discrete method, non-orthogonality boundary, nonlinear boundary value

一氧化碳作用下铁对一氧化氮的催化还原实验与动力学过程分析=**Experimental and Kinetics Process Analysis of NO Catalytic Reduction by Iron under the Action of CO** [刊, 汉] / ZHOU Hao-sheng, LU Ji-dong, ZHOU Hu, et al (National Key Lab of Coal Combustion under the Huazhong University of Science and Technology Wuhan, China, Post Code: 430074) // Journal of Engineering for Thermal Energy &Power. —2002, 17(1). —86~89

An experiment and analysis was conducted of the catalytic reduction process of NO by iron under the action of CO. It has been found that at a temperature of 1123 K the conversion rate of NO to N_2 was 70%. A very porous structure resulted after a reaction of NO with Fe and CO. An analysis indicates that under high temperatures the absorption ability of NO on iron oxides is stronger than that of CO, resulting in the presence of iron oxides on the reaction surface. It is assumed that the reaction interface in the reaction process was decided by the slower reaction rate between Fe and NO or between iron oxides and CO. On this basis set up preliminarily was a physical and mathematical model for the above reaction. **Key words**: nitric oxide, iron, catalytic reaction, kinetics

中小型煤粉炉的运行优化=**Optimized Operation of Medium and Small-sized Pulverized Coal-fired Boilers** [刊, 汉] / LU Ze-hua, XU Chun-hui (Thermal Energy Engineering Department, Tsinghua University, Beijing, China, Post Code: 100084) // Journal of Engineering for Thermal Energy &Power. —2002, 17(1). —90~92

The enhancement of operating efficiency of medium and small-sized pulverized coal-fired boilers with their operation controlled at an optimum air-coal ratio has always been a difficult issue. This comes about because the optimum air-coal ratio varies with boiler conditions, boiler load and ranks of coal fired and assumes a non-steady magnitude. Proceeding from the reverse balance method of boiler efficiency calculation and by taking furnace outlet temperature and exhaust gas temperature as major factors the authors have derived by logical reasoning the thermal efficiency judgement criteria for seeking an optimum air-coal ratio. On this basis a knowledge base was set up by a self-study system. As a result, a two-dimensional fuzzy decision table can be obtained. It has the boiler load and coal rank serving as parametric variables and the oxygen content of flue gas, which characterizes the optimum air-coal ratio, serving as dependent variables. This approach has solved the difficult problem of how to attain the high-efficiency operation of medium and small-sized pulverized coal-fired boilers during their on-line and real-time control. **Key words**: optimum air-coal ratio, thermal efficiency judgement criteria, fuzzy decision table, optimized operation

温度场分析的自适应有限元方法=**Self-adaptive Finite Element Method for the Analysis of a Temperature Field** [刊, 汉] / WANG Zhang-qi, AN Li-qiang (Mechanical Engineering Department, North China Electric Power University, Baoding, Hebei Province, China, Post Code: 071003) // Journal of Engineering for Thermal Energy &Power. —2002, 17(1). —92~94

Self-adaptive finite element method has its important application value in the analysis of practical engineering problems. In the light of the specific features of a temperature field the authors have presented a simple method for the estimation of local errors in the finite element calculation of a temperature field. In conjunction with Delaunay triangulation method of finite element mesh generation a study was conducted of the self-adaptive method for the finite element analysis of a temperature field. In addition, the error estimation of the temperature field and other functions, such as automatic division of

finite element mesh and the introduction of encryption, have been realized in a finite element analysis software for the rotor temperature field of a steam turbine. As a result, finite element numerical calculation results were obtained of a temperature field of constant precision. The effectiveness of the method recommended by the authors can be proved by the finite element mesh used for the analysis of the temperature field of a Chinese-made 300 MW steam turbine rotor and the relevant calculation results. **Key words**: finite element, mesh division, self-adaptation, temperature field

热电联产热、电按质分摊法的热量平衡＝**Thermal Balance of Heat and Electricity of a Cogeneration Plant by Using the Quality-based Method of Apportionment** [刊, 汉] / JING You-yin (North China Electric Power University, Baoding, Hebei Province, China, Post Code: 071003) // Journal of Engineering for Thermal Energy & Power. —2002, 17(1). —95 ~ 96

The author refers to a paper entitled "Establishment of a mathematical model for the quality-based method of apportionment of heat and electricity produced by a cogeneration plant and the relevant correction method". In this connection an analysis was conducted of the thermal balance of heat and electricity of a cogeneration plant by using the quality-based method of apportionment. Such an analysis has further demonstrated the accuracy and rationality of the quality-based method of apportionment. **Key words**: cogeneration of heat and electricity, factor of insufficient enthalpy drop of extracted steam, cold source loss of thermo-chemical power generation, quality-based method of apportionment

吉林镍业公司工业锅炉排污系统的改造＝**Modification of the Blow-down System of Industrial Boilers in Jilin Nickel Industry Co.** [刊, 汉] / BI Qing-sheng, SHANG Fu-min, SUN Shi, SHI Jiu-sheng (Energy Engineering Department, Changchun Engineering Institute, Changchun, China, Post Code: 130012) // Journal of Engineering for Thermal Energy & Power. —2002, 17(1). —97 ~ 98

On the basis of an analytical study of the blow-down system of four 20 t/h industrial boilers the system as a whole underwent an upgrading from the perspective of energy saving. In addition, a new type of extraction-evaporative flashing cooler has been studied and developed, which may provide certain reference data for the energy saving-oriented modification of blow-down systems of Chinese-made industrial boilers. **Key words**: industrial boiler, blow-down system, energy saving-oriented modification

提高 SZWP4—1.25—AⅡ＝**Enhancement of Power Output of Model SZWP-1.25-AⅡ Steam Boiler** [刊, 汉] / LI Shun-hai, LI yan (Neihe Municipal Boiler Inspection Institution, Nehe, Heilongjiang Province, China, Post Code: 161300) // Journal of Engineering for Thermal Energy & Power. —2002, 17(1). —99 ~ 100

After analyzing the cause of the excessively low output of Model SZWP4-1.25-AⅡ steam boiler the author has put forward a series of design modification measures, which have proved quite effective and may also be of certain reference value for the modification of similar type of boilers. **Key words**: Model SZWP boiler, power output, upgrading