

# 修正的 $k-\epsilon-k_p$ 双流体模型用于模拟旋流突扩燃烧室内气固两相流动

李志强, 魏 飞, 李荣先, 周力行  
(清华大学 化学工程系, 北京 100084)

**摘 要:** 对标准的  $k-\epsilon-k_p$  模型中  $\epsilon$  方程的源项进行了修正, 用于模拟有旋突扩气固两相湍流流动, 并与实验结果进行了比较, 得出的修正模型能够较好地模拟旋流流动, 对工程优化和设计具有指导意义。

**关 键 词:** 旋流流动; 修正的  $k-\epsilon-k_p$  双流体模型; 数值计算

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## 1 引 言

关于旋流气固两相流动的数值模拟, 早期多采用标准的  $k-\epsilon$  模型或代数应力模型 (ASM) 来模拟气相流场, 用随机轨道模型来模拟颗粒相, 这种模拟方法在工程研究中仍有广泛的应用。Boysan 和 Swithenbank 等首次用 ASM 模型加随机轨道模型模拟了旋风分离器内的二维强旋气粒两相流动, 预报结果给出了分离器内的气相轴向及切相速度、各个方向的湍流度、颗粒轨道、收集效率和阻力等, 其总体效果和实验数据大致符合<sup>[1]</sup>。张健在模拟带中心管多股进气的涡旋燃烧炉内的气固两相流动时, 分别用标准的  $k-\epsilon$  模型、ASM 模型和改进的 ASM 模型来模拟气相湍流, 结果表明: 与其它两个模型相比, 改进的 ASM 模型能够反应强旋流动的各向异性特点<sup>[2]</sup>。

当采用轨道模型来模拟颗粒流动时, 其缺点是难以给出颗粒数密度及速度分布, 因而颗粒相计算结果无法用实验结果来检验。周力行等根据拟流体的概念, 提出了颗粒湍动能输运方程模型 ( $k_p$  模型), 文献[3]用  $k-\epsilon-k_p$  双流体模型模拟了旋流数为 0.47 的突扩旋流两相流动, 其结果与实验符合较好, 能够给出颗粒的数密度和速度分布的详细信息。由于  $k-\epsilon-k_p$  模型仍采用了基于 Boussinesq

假设的各向同性的涡粘性系数, 不适于高度各向异性的强旋湍流流动和其它类型的复杂<sup>[1]</sup>流动, 但其计算量小、应用方便的优点使其在工程优化和设计方面仍有广阔的应用前景。文中针对  $k-\epsilon-k_p$  没有考虑流线弯曲和旋转效应的缺点, 用 Launder 提出的采用湍流时间尺度形式的 Richardson 数对  $\epsilon$  方程的源项进行修正, 使其适用于模拟强旋流动, 并将计算结果和实验做了比较。

## 2 修正的 $k-\epsilon-k_p$ 双流体模型

双流体模型就是在欧拉坐标系下求解气粒两相各自的质量守恒和动量守恒方程组, 在柱坐标系下三维湍流两相流动的稳态控制方程组可表达为如下通用形式。

气相控制方程组的一般表达式为:

$$\begin{aligned} & \frac{\partial}{\partial x}(\rho u \varphi) + \frac{\partial}{r \partial r}(r \rho v \varphi) + \frac{\partial}{r \partial \theta}(\rho w \varphi) \\ &= \frac{\partial}{\partial x} \left[ \Gamma_{\varphi} \frac{\partial \varphi}{\partial x} \right] + \frac{\partial}{r \partial r} \left[ r \Gamma_{\varphi} \frac{\partial \varphi}{\partial r} \right] + \frac{\partial}{r^2 \partial \theta} \left[ \Gamma_{\varphi} \frac{\partial \varphi}{\partial \theta} \right] + \\ & S_{\varphi} + S_{\varphi p} \end{aligned}$$

其中:  $\varphi$ 、 $\Gamma_{\varphi}$ 、 $S_{\varphi}$ 、 $S_{\varphi p}$  的具体表达式由表 1 给出。

颗粒相控制方程组的一般表达式为:

$$\begin{aligned} & \frac{\partial}{\partial x}(\rho_p u_p \varphi_p) + \frac{\partial}{r \partial r}(r \rho_p v_p \varphi_p) + \frac{\partial}{r \partial \theta}(\rho_p w_p \varphi_p) \\ &= \frac{\partial}{\partial x} \left[ \Gamma_{\varphi p} \frac{\partial \varphi_p}{\partial x} \right] + \frac{\partial}{r \partial r} \left[ r \Gamma_{\varphi p} \frac{\partial \varphi_p}{\partial r} \right] + \\ & \frac{\partial}{r^2 \partial \theta} \left[ \Gamma_{\varphi p} \frac{\partial \varphi_p}{\partial \theta} \right] + S_{\varphi p} + S_{\varphi p g} \end{aligned}$$

其中:  $\varphi_p$ 、 $\Gamma_{\varphi p}$ 、 $S_{\varphi p}$ 、 $S_{\varphi p, g}$  的具体表达式由表 2 给出。两相方程组中的  $\varphi$  和  $\varphi_p$  为求解变量,  $\Gamma_{\varphi}$  和  $\Gamma_{\varphi p}$  为扩散系数,  $S_{\varphi}$ 、 $S_{\varphi p}$ 、 $S_{\varphi p, g}$  分别为两相相互作用的源项。

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作者简介: 李志强(1970-), 男, 山西长治人, 清华大学博士后。

表 1 气相控制方程组中各项含义

$\varphi$	$\Gamma_\varphi$	$S_\varphi$	$S_{\varphi p}$
连续方程	1	0	$-\sum_p n_p \dot{m}_p$
x-方向动量方程	$u$	$\mu_e \left[ -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu_e \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial r} \left( \mu_e \frac{\partial u}{\partial r} \right) \right] + \rho g_x$	$-\sum_p \frac{\rho_p}{\sigma_p} (u - u_p)$ $-\nu \sum_p n_p \dot{m}_p$
r-方向动量方程	$v$	$\mu_e \left[ -\frac{\partial p}{\partial r} + \frac{\partial}{\partial x} \left( \mu_e \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial r} \left( \mu_e \frac{\partial v}{\partial r} \right) \right] + \rho g_r - \frac{2\mu_e}{r} \frac{\partial w}{\partial \theta}$	$-\sum_p \frac{\rho_p}{\sigma_p} (v - v_p)$ $-\nu \sum_p n_p \dot{m}_p$
$\theta$ -方向动量方程	$w$	$\mu_e \left[ \frac{\partial}{\partial r} \left( \mu_e \left( \frac{\partial w}{\partial r} - \frac{v}{r} \right) \right) + \frac{\partial}{\partial \theta} \left( \mu_e \left( \frac{\partial w}{\partial \theta} + 2v \right) \right) \right] + \rho g_\theta + \frac{\mu_e}{r} \left( \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w}{r} \right) - \frac{\rho w}{r}$	$-\sum_p \frac{\rho_p}{\sigma_p} (w - w_p)$ $-\nu \sum_p n_p \dot{m}_p$
湍动能 $k$	$k$	$\frac{\mu_e}{\sigma_k} G_k - \epsilon$	$G_p - k \sum_p n_p \dot{m}_p$
湍动能耗散率 $\epsilon$	$\epsilon$	$C_1 G_k \left( \frac{\epsilon}{k} \right) - C_2 (1 - C_{gs} R_{igs}) \frac{\rho \epsilon^2}{k}$	$\frac{\epsilon}{k} C_1 (G_p - k \sum_p n_p \dot{m}_p)$

表 2 颗粒相控制方程组中各项含义

$\varphi$	$\Gamma_\varphi$	$S_\varphi$	$S_{\varphi, s}$
颗粒似密度连续方程	1	$\frac{\dot{m}_p}{\sigma_p}$	0
x-方向动量方程	$u_p$	$\mu_p \left[ \frac{\partial}{\partial x} \left( \mu_p \frac{\partial u_p}{\partial x} \right) + \frac{\partial}{\partial r} \left( \mu_p \frac{\partial u_p}{\partial r} \right) \right] + \rho_p g_x + \frac{\partial}{\partial r} \left( \mu_p \frac{v_p}{\sigma_p} \frac{\partial \rho_p}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \mu_p \frac{w_p}{\sigma_p} \frac{\partial \rho_p}{\partial \theta} \right) + \frac{\partial}{\partial r} \left( \mu_p \frac{v_p}{\sigma_p} \frac{\partial \rho_p}{\partial x} \right) + \frac{\partial}{\partial \theta} \left( \mu_p \frac{w_p}{\sigma_p} \frac{\partial \rho_p}{\partial x} \right)$	$\frac{\rho_p}{\sigma_p} (u - u_p)$ $u_p \dot{m}_p$
r-方向动量方程	$v_p$	$\mu_p \left[ \frac{\partial}{\partial x} \left( \mu_p \frac{\partial v_p}{\partial x} \right) + \frac{\partial}{\partial r} \left( \mu_p \frac{\partial v_p}{\partial r} \right) \right] + \rho_p g_r - \frac{2\mu_p}{r} \frac{\partial w_p}{\partial \theta} + \frac{\partial}{\partial r} \left( \mu_p \frac{v_p}{\sigma_p} \frac{\partial \rho_p}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \mu_p \frac{w_p}{\sigma_p} \frac{\partial \rho_p}{\partial r} \right) + \frac{\partial}{\partial x} \left( \mu_p \frac{v_p}{\sigma_p} \frac{\partial \rho_p}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \mu_p \frac{w_p}{\sigma_p} \frac{\partial \rho_p}{\partial r} \right)$	$\frac{\rho_p}{\sigma_p} (v - v_p)$ $v_p \dot{m}_p$
$\theta$ -方向动量方程	$w_p$	$\mu_p \left[ \frac{\partial}{\partial r} \left( \mu_p \left( \frac{\partial w_p}{\partial r} - \frac{v_p}{r} \right) \right) + \frac{\partial}{\partial \theta} \left( \mu_p \left( \frac{\partial w_p}{\partial \theta} + 2v_p \right) \right) \right] + \rho_p g_\theta + \frac{\mu_p}{r} \left( \frac{\partial v_p}{\partial \theta} + \frac{\partial w_p}{\partial r} - \frac{w_p}{r} \right) - \frac{\rho_p w_p}{r}$	$\frac{\rho_p}{\sigma_p} (w - w_p)$ $w_p \dot{m}_p$
颗粒相湍动能方程	$k_p$	$G_{pk} + \frac{\partial}{\partial x} \left( k_p \frac{v_p}{\sigma_p} \frac{\partial \rho_p}{\partial x} \right) + \frac{\partial}{\partial r} \left( k_p \frac{v_p}{\sigma_p} \frac{\partial \rho_p}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( k_p \frac{v_p}{\sigma_p} \frac{\partial \rho_p}{\partial \theta} \right)$	$G_{gk}$

表 1 和表 2 中的,  $G_k$ 、 $G_p$ 、 $G_{pk}$ 、 $G_{gk}$ 、 $\mu_e$ 、 $\mu_p$ 、 $\nu$  和  $\nu_p$  的表达式见文献[4]。 $\epsilon$  方程中的附加项  $C_{gs}R_{igs}$  为本文所采用的 P.J.Smith 修正式, 在该修正式中充分考虑了旋转及流线弯曲效应, 以便提高旋流流动预报的合理性, 其中湍流时间尺度为:

$$R_{igs} = \frac{k^2}{\epsilon^2} \frac{w}{r^2} \frac{\partial}{\partial r} (rw) = \frac{k^2}{\epsilon^2} \frac{w}{r} \left( \frac{\partial w}{\partial r} + \frac{w}{r} \right),$$

$$C_{gs} = 0.001 \sim 0.2, \text{文中 } C_{gs} = 0.015$$

### 3 模拟对象及数值解法

为了检验该修正的双流体模型, 用它来模拟旋流突扩气固两相流动, 并与相位多普勒颗粒分析仪 (PDPA) 所测量的结果进行对比, 以便对该模型进行评价。

模拟的几何结构如图 1 所示, 其中 1 为带颗粒的直流射流(颗粒为 50  $\mu\text{m}$  的玻璃微珠), 2 为旋流射流, 3 为直流射流。为保证湍流射流的充分发展, 计算区域的轴向长度取为 4 m。

在数值求解时对气相和颗粒相均采用有限差分数值解法, 采用交错网格和上风差分格式, 对差分方程采用  $p-v$  修正的 SIMPLE 算法和 TDMA 逐行逐

面迭代求解, 网格节点取为  $69 \times 50 \times 11$ , 为了计算精度, 采用非均匀性网格, 在进口和壁面附近加密网格。计算中的迭代收敛精度气相取质量余源和小于  $1 \times 10^{-3}$ , 颗粒相取质量余源和小于  $1 \times 10^{-2}$ 。各物理量均取均匀性进口条件; 出口取为充分发展流条件:  $\partial \varphi / \partial x = 0$ , ( $\varphi = u, v, w, p, k, \epsilon$ ); 轴线上为对称性条件:  $\partial \varphi / \partial x = 0$ , ( $\varphi = u, v, w, p, k, \epsilon$ ),  $v = 0, w = 0$ ; 壁面处: 气相取无滑移条件, 颗粒法向速度分量为零, 其它速度分量的梯度和浓度梯度为零。

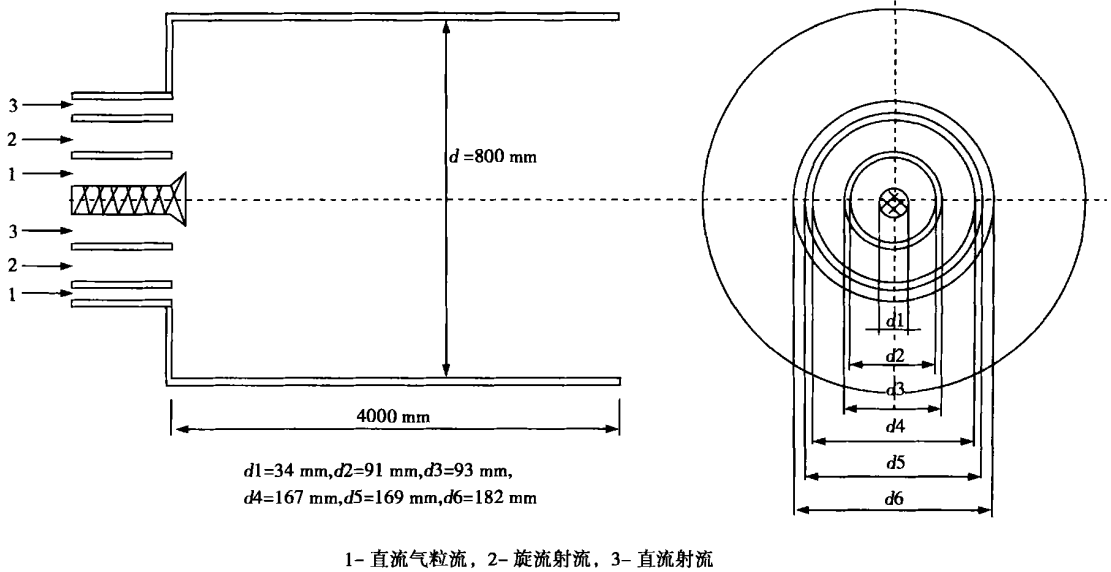


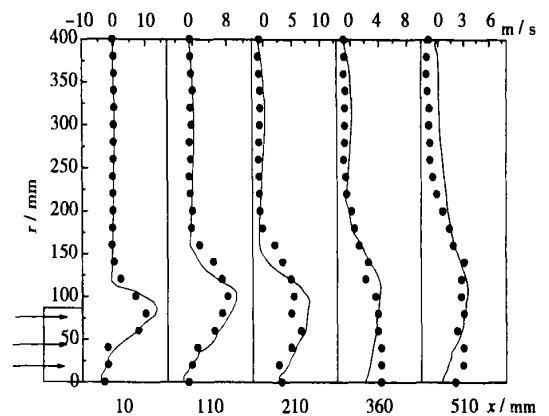
图 1 计算模型的几何尺寸

#### 4 模拟结果及分析

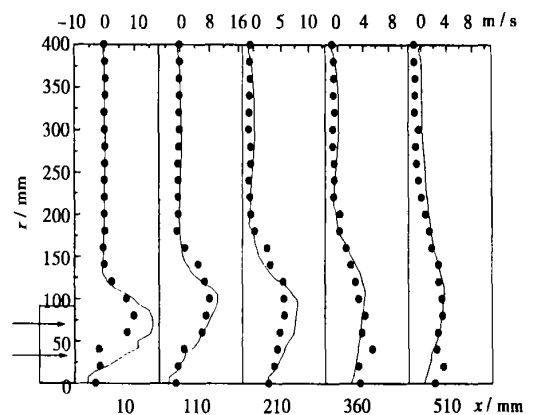
旋流突扩燃烧室气固两相流的进口参数是: 一次风速 12.05 m/s, 一次风质量流率 19.76 g/s; 旋流二次风速 16.0 m/s, 旋流二次风质量流率 70.8 g/s; 旋流二次风切向速度 14.0 m/s; 直流二次风速 16.07 m/s, 直流二次风质量流率 16.85 g/s; 颗粒平均粒径 50  $\mu\text{m}$ ; 颗粒质量流率 3.1855 g/s; 固气比 0.02966; 颗粒材料密度 2200  $\text{kg}/\text{m}^3$ 。

由预报的冷态气相和颗粒相的轴向速度分布 (见图 2) 可以看出预报的结果和实验值吻合的比较好, 只是在进口射流的扩展角内有点误差。预报和实验均表明在进口中心轴线附近存在一个中心回流区, 这符合旋流流动的机理, 只是所预报的中心回流区比实际测量的要长而窄, 比较窄的原因可能仍是  $k-\epsilon-k_p$  模型的各向同性所造成。而比较长可能是对  $\epsilon$  方程的修正所致, 所以对  $k-\epsilon-k_p$  模型的修正仍需作进一步的研究。

图 3 所示为两相切向速度分布, 可以看出, 两相切向速度的预报值和实验值基本符合, 都呈现出旋流流动中典型的 Rankine 涡结构。但是预报的切向速度在似固核内峰值附近大于实验值, 而在自由涡区小于实验值。这可能是  $k-\epsilon-k_p$  模型模拟旋流的本身缺陷所造成, 但也不排除实验误差的原因, 这可从中心轴线上的实验值不为零所证实, 而预报值



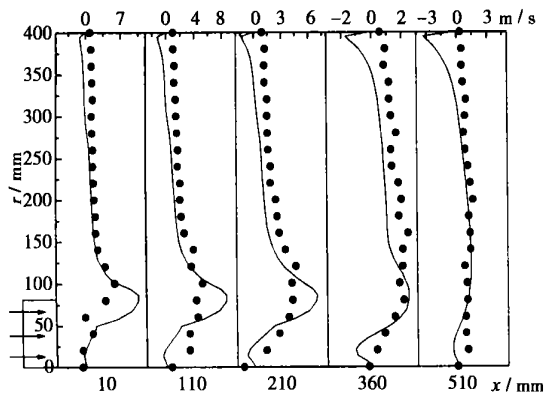
(a) 气相轴向平均速度分布图



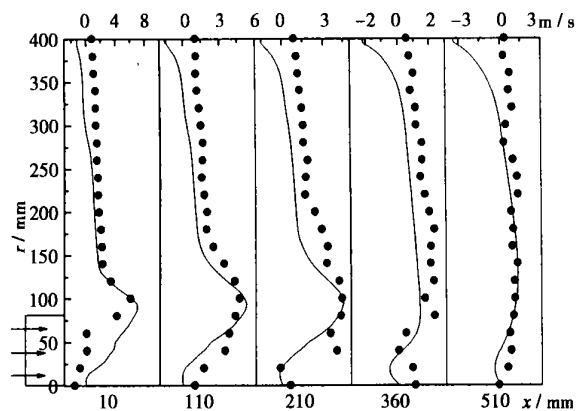
(b) 颗粒相轴向平均速度分布图

图 2 两相轴向平均速度分布图

在中心轴线上的值为零,可以说预报值在此处是合理的。



(a) 气相切向平均速度分布图



(b) 颗粒相切向平均速度分布图  
——计算值, ● 实验值

图 3 两相切向平均速度分布图

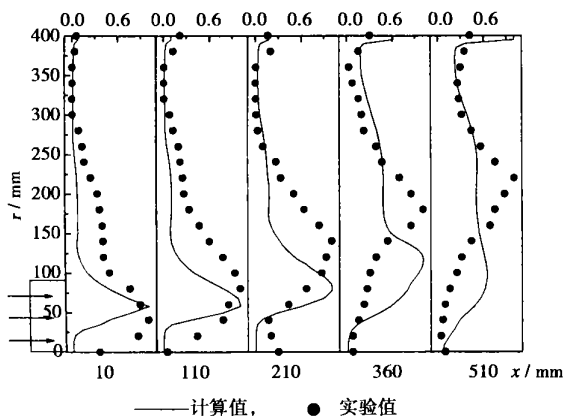


图 4 颗粒相对数密度分布图

颗粒相对数密度分布如图 4 所示,可以看出,预报值和实验值在  $x = 10 \text{ mm}$ 、 $x = 110 \text{ mm}$  和  $x = 210 \text{ mm}$  三个测量截面上大致符合,并且模拟结果表明在壁面附近颗粒浓度较高,这与实验结果及旋流机理都是相符合的。随着离心力和湍流扩散的作用,浓度峰值逐渐向边壁移动,在  $x = 360 \text{ mm}$  和  $x = 510 \text{ mm}$  两个截面上预报值和实验值出入较大。究其原因,一个是实验方面,三维相位多普勒颗粒分析仪(PDPA)不能直接测量颗粒浓度分布,而是在测量出颗粒速度和粒径后通过软件计算得到。PDPA 系统在测量颗粒速度和粒径时的误差将会使浓度的误差变得更大。另一个原因是  $k-\epsilon-k_p$  模型本身的缺陷所造成的计算误差。故总体认为浓度预报值有一定的合理性,能够用来指导工程设计。

### 5 结 论

(1) 采用 P. J. Smith 修正式修正的  $k-\epsilon-k_p$  双流体模型可以合理地预报旋流突扩气固两相流场,预报的轴向平均速度与 PDPA 测量的实验值符合很好;预报的切向速度比实验值有一定的合理性;预报的颗粒浓度分布与实验值基本吻合,但还有一定的差距,需要从实验和数学模型两方面进行更深入的研究。

(2) 计算结果和实验结果均表明进口中心轴线附近存在一个中心回流区,与旋流流动的机理是符合的。

(3) 两相切向速度分布均存在典型的 Rankine 涡结构。

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**Diffusion in a Gas-solid Two-phase Turbine** [刊, 汉] / LU Jia-hua, LING Zhi-guang (College of Aviation Transportation under the Shanghai University of Engineering & Technology, Shanghai, China, Post Code: 200336) // Journal of Engineering for Thermal Energy & Power. — 2003, 18(5). — 454 ~ 458

In the totally three-dimensional viscous turbulent flow field of a dual-stage transonic gas turbine a Lagrangian numerical simulation was respectively conducted of the turbulent diffusion movement of coal particles of different particle sizes and mixed coal particles of 5—50  $\mu\text{m}$  distribution diameter (volume fraction =  $0.903 \times 10^{-4}\%$ ) in a blade flow path. This was accomplished with the use of a gas-solid two-way coupled model. As a result, the corresponding movement trajectories and slippage were obtained. As compared to the results of numerical simulation under the assumed condition of non-viscid, laminar flow and one-direction coupling, more realistic movement characteristics of particles were secured. It should be specially noted that at the blade pressure surface, blade root and tip the distribution of impact-point of particles with blades is in relatively good agreement with the distribution condition of actual blade impact/erosion points. This has confirmed the non-negligible effect of the turbulent diffusion characteristics of particles in the blade flow path. The results of the numerical simulation will provide a more reliable basis for furthering the optimization of the aerodynamic design of gas-solid two-phase turbine blades. **Key words:** gas-solid two-phase turbine, viscous turbulent flow field, turbulent diffusion, two-way coupling, Lagrangian method, numerical simulation

**修正的  $k-\varepsilon-k_p$  双流体模型用于模拟旋流突扩燃烧室内气固两相流动 = Numerical Simulation of Gas-solid Two-phase Flows in a Swirling-flow Combustor through the Use of a Modified  $k-\varepsilon-k_p$  Two-fluid Model** [刊, 汉] / LI Zhi-qiang, WEI Fei, LI Rong-xian, et al (Department of Chemical Engineering, Tsinghua University, Beijing, China, Post Code: 100084) // Journal of Engineering for Thermal Energy & Power. — 2003, 18(5). — 459 ~ 462

The source item of  $\varepsilon$  equation in a standard  $k-\varepsilon-k_p$  model after a modification is used to simulate swirling gas-solid two-phase turbulent flows. The simulation results were compared with experimental data. The resulting modified model can relatively well simulate the swirling flows and play a significant role in promoting an optimized engineering design. **Key words:** swirling flow, modified  $k-\varepsilon-k_p$ , two-fluid model, numerical simulation

**有再循环系统的超音速两相流升压性能的研究 = A Study on the Performance of a Supersonic Steam-liquid Two-phase Flow Pressure-boosting Unit Equipped with a Hot-water Recirculation System** [刊, 汉] / GUO Ying-li, LI sheng, YAN Jun-jie, et al (College of Energy and Power Engineering under the Xi'an Jiaotong University, Xi'an, China, Post Code: 710049) // Journal of Engineering for Thermal Energy & Power. — 2003, 18(5). — 463 ~ 466

A theoretical calculation and experimental study is conducted of a supersonic steam-liquid two-phase flow pressure-boosting unit, which incorporates a hot-water recirculation system. It has been found that under a condition of constant inlet steam parameters the pressure boosting performance will decrease with an increase in feed water temperature and increase with an increase in outlet water flow. The outlet water temperature will decrease with an increase in outlet water flow, and increase with a rise in feedwater temperature. The unit under discussion not only retains the self-adaptive characteristics of the original outlet pressure, but also features self-adaptive characteristics of outlet flow rate. **Key words:** supersonic, steam-liquid two-phase flow, shock wave, constant flow rate characteristics

**一种湿法烟气脱硫方式的试验研究 = Experimental Research of a Wet-process Flue-gas Desulfurization Method** [刊, 汉] / SAI Jin-Cong, WU Shao-hua, WANG Hong-tao (College of Energy Science & Engineering under the Harbin Institute of Technology, Harbin, China, Post Code: 150001) // Journal of Engineering for Thermal Energy & Power. — 2003, 18(5). — 467 ~ 470

A flue gas desulfurization method based on the use of a groove-shaped flow-guide core and flue-gas transverse sweep is proposed, which has been studied on a small-size test rig. A qualitative analysis was conducted of the impact of flue gas flow speed, temperature and gas-liquid contact area on flue-gas desulfurization efficiency. The slag formation problem of the test rig is also briefly analyzed and discussed. **Key words:** transverse sweep of gases, groove-type core, wet-process flue-gas desulfurization, slag formation