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# 二阶全展开 ETG 有限元方法在方腔自然对流模拟中的应用

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摘　要: 应用二阶全展开 ETG 有限元方法离散求解 $N-S$ 方程和能量方程, 并以零初值方腔自然对流问题为例进行了数值模拟。计算了不同瑞利数条件下方腔自然对流的流场和温度场, 最终达到的稳态结果与标准数值解符合很好, 并且较好地反映了流场和温度场的时间演化过程, 特别是捕捉到了分叉前后流场中涡结构的变化。结果表明二阶全展开 ETG 有限元方法有较好的稳定性和较高的精度, 在计算温度场和流场的时间演化过程方面有一定优点。

关　键　词: 二阶全展开 ETG 有限元; 方腔; 自然对流

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## 1　引　言

在计算传热学领域内, 关于自然对流换热的数值模拟已经取得了许多成果。一般来说, 定常条件下平均温度场和平均速度场的计算在一定的瑞利数和雷诺数范围内较有把握, 而对于非定常条件下或因大尺度分叉导致的时变过程的计算, 特别是分叉点的确定, 仍存在许多有待研究的问题。笔者等发展的二阶全展开 ETG 有限元方法应用于计算流场中大尺度涡的时间演化过程取得了较好的效果[1-3]。本文拟将这一方法推广于自然对流换热的数值模拟, 重点在于反映其时变过程, 特别是涡结构发生变化时的瞬刻状态, 并以计算传热经常用作"标模"的方腔零初值低瑞利数的自然对流换热为例, 来检验其有效性。

## 2　控制方程及数值方法

### 2.1　基本方程

分别取无因次长度、时间、速度、压力和温度为 $\frac{x_i^*}{H}$、$\frac{t^*}{H^2/\alpha}$、$\frac{u_i^*}{\alpha/H}$、$\frac{p^*}{\rho(\alpha/H)^2}$ 和 $\frac{T^*-T_L}{T_R-T_L}$, 则方腔自然对流基本方程的无因次形式:

$$\frac{\partial u_i}{\partial x_i}=0 \tag{1}$$

$$\frac{\partial u_i}{\partial t}+\frac{\partial(u_i u_j)}{\partial x_j}=-\frac{\partial p}{\partial x_i}+Pr\frac{\partial^2 u_i}{\partial x_j\partial x_j}+RaPrT\delta_{ij} \tag{2}$$

$$\frac{\partial T}{\partial t}+\frac{\partial(u_j T)}{\partial x_j}=\frac{\partial^2 T}{\partial x_j\partial x_j} \tag{3}$$

式中: $\rho$－密度; $u_i$－速度; $p$－压力; $T$－温度; $H$－特征长度; $Pr$－普朗特数; $Ra=\frac{g\beta H^3(T_R-T_L)}{\alpha\nu}$ 为瑞利数, 其中 $T_L$ 和 $T_R$ 分别为方腔左右侧的温度, $\alpha$ 为对流换热系数, $\beta$ 为膨胀系数, $\nu$ 为运动粘度; 带星号的量为有因次物理量, 不带星号的量为无因次物理量。

### 2.2　数值方法

本文采用笔者等所发展的二阶全展开 ETG 有限元方法对方程进行离散, 通过对式(2)和(3)中的时变项进行 Taylor 展开, 并把时间导数用空间导数来代替, 其作用相当于引入了人工粘性。对于 $N$-$S$ 方程(2)的离散与文献[1]类似, 此处仅给出离散能量方程的过程。采用二阶全展开 ETG 有限元法处理式(3), 整理后可得:

$$\left\{1-\frac{1}{2}\Delta t\frac{\partial^2}{\partial x_i\partial x_i}\right\}\frac{(T^{n+1}-T^n)}{\Delta t}=\left\{1-\frac{1}{2}\Delta t u_j^n\frac{\partial}{\partial x_j}\right\}\left\{-u_j^n\frac{\partial T^n}{\partial x_j}+\frac{\partial^2 T^n}{\partial x_j\partial x_j}\right\}-\frac{1}{2}\left(u_j^n-u_j^{n-1}\right)\frac{\partial T^n}{\partial x_j} \tag{4}$$

采用标准 Galerkin 方法, 有限元积分表达式可化为:

$$\int_{\Omega_e}\left\{1-\frac{1}{2}\Delta t\frac{\partial^2}{\partial x_i\partial x_i}\right\}\frac{(T^{n+1}-T^n)}{\Delta t}N_I\mathrm{d}\Omega=\int_{\Omega_e}\left(1-\frac{1}{2}\Delta t u_j^n\frac{\partial}{\partial x_j}\right)\left(-u_j^n\frac{\partial T^n}{\partial x_j}+\frac{\partial^2 T^n}{\partial x_j\partial x_j}\right)\times N_I d\Omega-\int_{\Omega_e}\frac{1}{2}\left(u_j^n-u_j^{n-1}\right)\frac{\partial T^n}{\partial x_j}N_I\mathrm{d}\Omega \tag{5}$$

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将:

$$T = N_J T_J^n \tag{6}$$

代入上式有:

$$A_{IJ} T_J^{n+1} = (B_{IJ} T_J^n + C_{IJ} T_J^n)\Delta t + A_{IJ} T_j^n \tag{7}$$

其中:

$$A_{IJ} = \int_{\Omega_e} N_I N_J \mathrm{d}\Omega + \int_{\Omega_e} \frac{1}{2}\Delta t \frac{\partial N_I}{\partial x_i}\frac{\partial N_J}{\partial x_i}\mathrm{d}\Omega \tag{8}$$

$$B_{IJ} = \int_{\Omega_e} \left(-N_I u_j^n \frac{\partial N_J}{\partial x_j} - \frac{\partial N_I}{\partial x_i}\frac{\partial N_J}{\partial x_i}\right)\mathrm{d}\Omega - \int_{\Omega_e} \frac{1}{2}\Delta t u_j^n u_j^n \frac{\partial N_I}{\partial x_j}\frac{\partial N_J}{\partial x_j}\mathrm{d}\Omega \tag{9}$$

$$C_{IJ} = \int_{\Omega_e} -\frac{1}{2}\left(u_j^n - u_j^{n-1}\right)\frac{\partial N_J}{\partial x_j} N_1 \mathrm{d}\Omega \tag{10}$$

以上完成了单元的分析, 在计算域内将所有单元的有限元方程进行迭加, 从而构成总体的有限元方程:

$$AT^{n+1} = (BT^n + CT^n)\Delta t + AT^n \tag{11}$$

上式这种离散格式隐含了流线迎风的耗散作用, 具有较高的精度和稳定性。

### 2.3　零初值方腔自然对流的数值模拟

方腔自然对流的几体形状及边界条件如图 1 所示, 计算采用的网格为 81× 81 的均匀网格, 普朗特数为 0.71, 瑞利数分别为 $10^3$、$10^4$、$10^5$ 和 $10^6$。

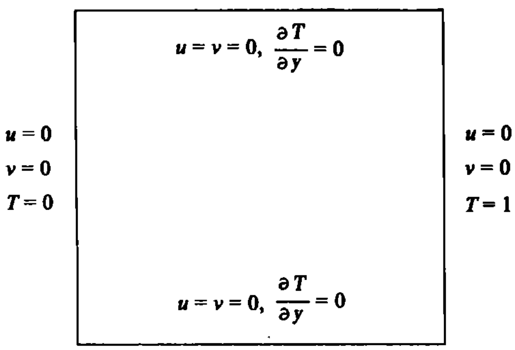


图 1　方腔自然对流的计算模型

表 1　主要参数对比表

| Ra | 结果 | $u_{max}$ | $y$ | $v_{max}$ | $x$ | $\overline{Nu}$ |
|---|---|---|---|---|---|---|
| $10^3$ | 本文 | 3.650 | 0.813 | 3.700 | 0.176 | 1.115 |
| | 标准解[4] | 3.649 | 0.813 | 3.697 | 0.178 | 1.118 |
| $10^4$ | 本文 | 16.186 | 0.824 | 19.608 | 0.122 | 2.248 |
| | 标准解[4] | 16.178 | 0.823 | 19.617 | 0.119 | 2.243 |
| $10^5$ | 本文 | 34.628 | 0.850 | 68.124 | 0.072 | 4.536 |
| | 标准解[4] | 34.722 | 0.855 | 68.590 | 0.066 | 4.519 |
| $10^6$ | 本文 | 64.845 | 0.852 | 219.433 | 0.041 | 8.825 |
| | 标准解[4] | 64.630 | 0.850 | 219.360 | 0.0379 | 8.800 |

## 3　结果分析

不同瑞利数下达到稳态时的流线图和等温线图分别如图 2 的(a)~(d)所示, 与标准数值解[4] 及其他学者[5-6] 计算得到的曲线符合得很好。表 1 中列出了达到稳态时垂直中心线上的最大水平速度及其位置、水平中心线上的最大垂直速度及其位置和空间平均 $Nu$ 数, 并与数值标准解进行了比较, 本文的计算结果与数值标准解符合较好, 表明本文所采用的二阶全展开 ETG 有限元格式具有较高的精度和稳定性。

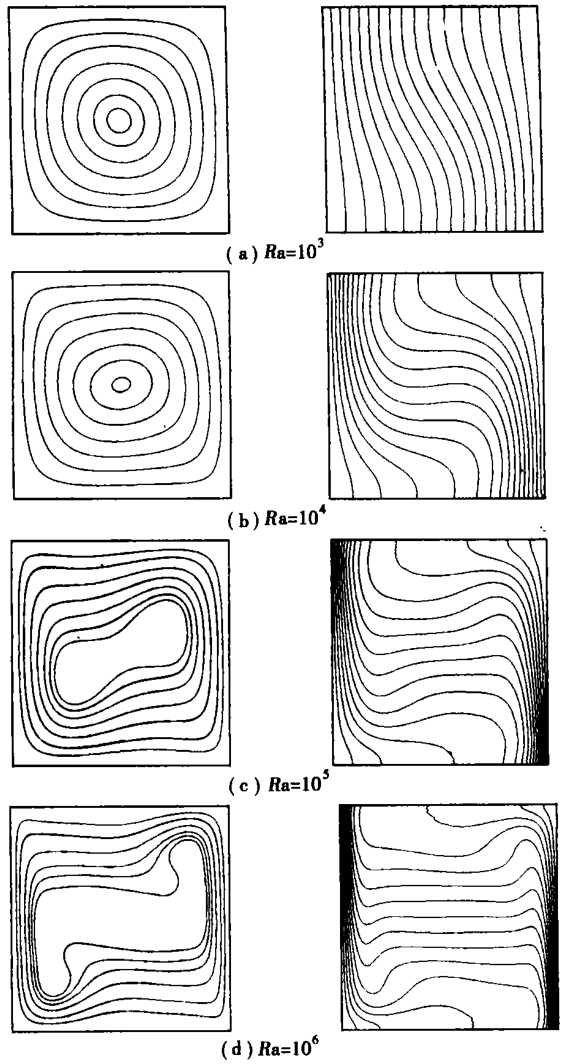


图 2　不同瑞利数下达到稳态时的流线图(左图)和等温线图(右图)

另外本文还给出了 $Ra= 10^6$ 条件下不同时刻的流线图和等温线图(图中标注时刻为无因次时间), 如图 3 所示, 从图中可以清晰地看到流场和温度场

的变化过程。

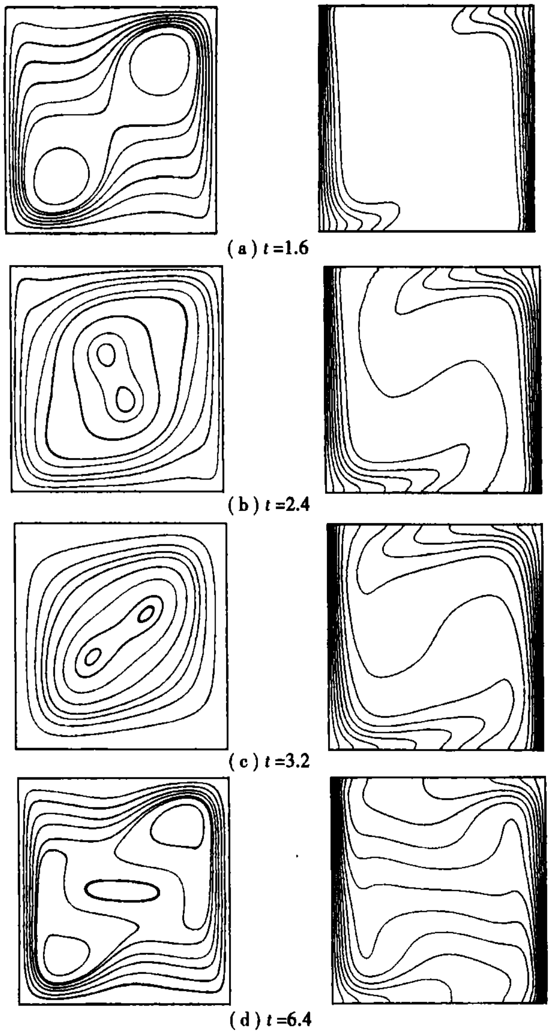


图 3　$Ra=10^6$ 不同时刻的流线图(左图)和等温线图(右图)

从图 3(c)和(d)中看出, 随着时间变化, 流场中的涡结构发生了变化, 由一对内涡迅速发展为三涡状态, 具体时间为 $t=4.5\sim4.6$, 这是低瑞利数下分叉的表现形式。由初始状态发展到最后的稳定状态, 流场中的涡结构要发生多次分叉, 采用二阶全展开 ETG 有限元方法均能捕捉到, 本文只给出这一阶段旋涡结构的瞬间变化, 如图 4 所示, 可以清晰地看到流场中两个大涡拉伸而产生第三个涡的过程。与前人的工作相比, 关于旋涡的时间演化及结构演化的细部描述方面, 二阶全展开 ETG 有限元方法具有明显优势。

## 4　结　语

(1) 采用二阶全展开 ETG 有限元方法对低瑞利数下的零初值方腔自然对流进行了数值模拟, 计算了不同瑞利数下的流场和温度场, 达到稳态时的流线图、等温线图及主要参数均与标准数值解符合较好, 表明二阶全展开 ETG 有限元方法在计算温度场和流场方面具有较高的稳定性和精度。

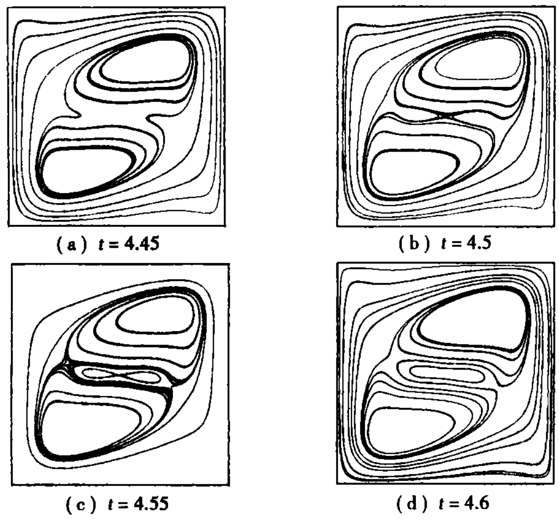


图 4　涡结构的变化

(2) 采用二阶全展开 ETG 有限元方法可以很好地反映温度场和流场的时变过程, 提供了丰富的温度场与流场信息, 并且捕捉到了流场中涡结构的变化细节。同时该方法与大涡模拟的结合, 特别适合于描述湍流温度场及流场中大尺度涡系的时间演化, 有关工作将另文阐述。

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was accomplished by the use of a gas-solid drag force. By way of simulations a transformation process was obtained of the generation, movement and explosion/cracking of gas bubbles, which has been found to be in full agreement with experimental results. By using different drag force models a simulation was performed of the fluidized bed dense two-phase flows. A comparison with Kuipers experimental results shows that the use of a Gidaspow drag force model can result in a more accurate description of the dense two-phase flows in the fluidized bed. **Key words**: Eulerian model, gas-solid two-phase flow, gas bubble, fluidized bed

环栅式动力除尘器的两相流数值模拟＝**Numerical Simulation of Two-phase Flows of a Cascade-ring Type of Aerodynamic Dust Collector** [刊, 汉] / LIN Feng, LIN Long (Harbin No. 703 Research Institute, Harbin, China, Post Code: 150036), HU Qi-di (Shenzhen Meishi Electric Power Industrial Co. Ltd., Shenzhen, China, Post Code: 518000), ZHANG Shi-lei (Thermal Power Plant of Jilin Petroleum Group Co. Ltd., Jilin, China, Post Code: 138000) //Journal of Engineering for Thermal Energy & Power. — 2004, 19(2). —134～136, 190

A numerical simulation of two-dimensional two-phase flows was conducted of a cascade-ring aerodynamic dust collector in an effort to enhance its performance through an improved design. During the simulation a particle trajectory model has been employed with a k-ε model being used to simulate gas-phase turbulent flows and a Stochas model used to describe the turbulent diffusion of a particle phase. The flow conditions of ash particles and gas, and the characteristics of the dust collector were studied and analyzed. **Key words**: two-phase flow, numerical simulation, dust removal, air purification

二阶全展开 ETG 有限元方法在方腔自然对流模拟中的应用＝**The Application of a Second-order and Full-extension ETG Finite Element Method for the Simulation of Natural Convection in a Square Cavity** [刊, 汉] / WEI Ying-jie (Astronautics Engineering and mechanics Department & Engineering under the Harbin Institute of Technology, Harbin, China, Post Code: 150090), HE Zhong-yi (School of Municipal & Environmental Engineering under the Harbin Institute of Technology, Harbin, China, Post Code: 150090) //Journal of Engineering for Thermal Energy & Power. — 2004, 19(2). —137～139

A second-order and full-extension ETG finite element method was employed to carry out a discrete solution for a N-S equation and an energy equation. With a square-cavity natural convection problem of zero initial value serving as an example a numerical simulation was conducted. The flow and temperature fields of natural convection in a square cavity at different Rayleigh numbers were calculated. The steady-state results being finally attained are in very good agreement with those of a standard numerical solution. Moreover, the time evolution process depicting the flow and temperature fields has been reflected quite well. Especially worth mentioning here is the capture of the change of vortex structure in the flow field before and after bifurcation. All the above shows that the second-order and full-extension ETG finite element method features a relatively fair stability and precision, and has its definite merits when used to evaluate the time evolution process of temperature and flow fields. **Key words**: second-order and full-extension ETG finite element, square cavity, natural convection

表面活性剂减阻流体湍流空间结构试验研究＝**Experimental Research on the Turbulent Spatial Structure of a Drag Reducing Fluid with a Surfactant being Added** [刊, 汉] / WANG De-zhong, HU You-qing, WANG Song-ping, ZHOU Rong-sheng (Institute of Mechanical and Power Engineering under the Shanghai Jiaotong University, Shanghai, China, Post Code: 200030) //Journal of Engineering for Thermal Energy & Power. — 2004, 19(2). —140～143

An experimental study was conducted of the turbulent flow field of a CTAC drag reducing fluid in a two-dimensional flow