

# 复杂激励下多级齿轮传动系统分岔特性及动载荷分析

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**摘要:** 为了研究复杂激励下多级齿轮传动系统的分岔特性及动载荷系数, 建立了包含时变啮合刚度、啮合阻尼、传递误差和齿侧间隙等因素的6自由度非线性动力学模型。采用4~5阶变步长Runge-Kutta法对系统无量纲动力学微分方程进行求解, 得出齿轮传动系统的分岔图及齿轮副动载荷系数变化曲线。计算表明: 随齿侧间隙、啮合刚度波动值的增加, 系统趋于不稳定, 经历了由单周期、多周期到混沌运动的状态, 动载荷系数呈增大趋势, 并出现脱啮现象; 当啮合阻尼比、扭转刚度增大时, 系统逐渐趋于稳定, 由混沌运动过渡到单周期运动, 动载荷系数逐渐减小, 齿轮副由双边冲击过渡到单边冲击或无冲击状态, 脱啮现象得到改善。

**关键词:** 齿轮传动; 非线性动力学; 分岔; 混沌; 动载荷系数

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## 引言

齿轮系统动力学问题包含时变刚度、侧隙、误差、阻尼等多种复杂的非线性因素<sup>[1~2]</sup>, 研究各种因素对系统动力学行为的影响对提高齿轮装置动态性能有着重要意义。近年来国内外学者对齿轮系统非线性动力学问题开展了大量的研究工作, 考虑时变啮合刚度、啮合阻尼、传递误差、齿侧间隙、齿面摩擦等因素<sup>[3~7]</sup>, 建立了齿轮系统非线性动力学数学模型或有限元模型, 应用解析法、Runge-Kutta法、谐波平衡法、Poincaré-Newton-Floquet法、有限元法等方法对非线性动力学模型进行求解<sup>[8~12]</sup>, 得到齿轮系统的分岔、混沌、冲击等非线性振动特性, 并研究了不同非线性因素对系统动态响应的影响。

上述文献的分析对象多为单级齿轮传动系统, 且仅考虑部分因素对系统非线性振动特性的影响。本研究综合考虑多种影响因素, 建立复杂激励下多

级齿轮传动系统非线性动力学模型, 采用4~5阶变步长Runge-Kutta法求解系统的无量纲动力学微分方程, 研究啮合刚度、啮合阻尼、齿侧间隙、扭转刚度等因素对齿轮系统分岔特性与动载荷系数的影响。

## 1 齿轮传动系统动力学模型

以某齿轮箱三级斜齿轮传动系统为研究对象, 采用集中参数法, 建立的扭转振动分析模型如图1所示。

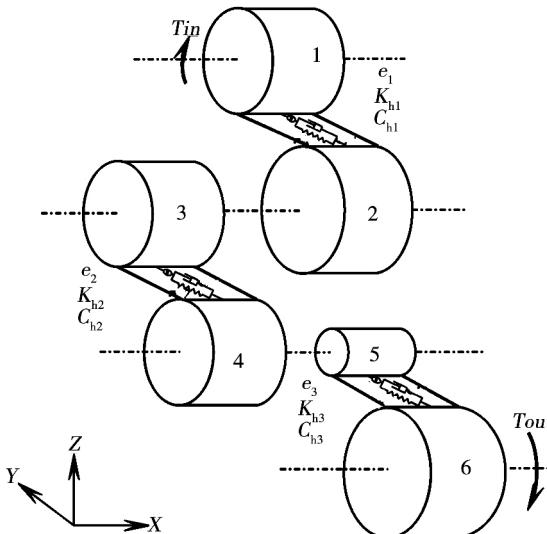


图1 齿轮传动系统动力学模型

Fig. 1 The dynamic model of gear transmission system

各斜齿轮副齿面啮合点间因振动和误差产生的沿啮合点法线方向的相对位移以及齿轮之间产生的扭转位移为:

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在  $0.01 - 0.06$  之间变化, 得到图 4 所示的系统位移 - 喷合阻尼比分岔图。当  $\xi$  较小时, 系统处于混沌状态; 当  $\xi > 0.029$  时, 系统由混沌运动直接进入单周期运动, 响应趋于稳定。

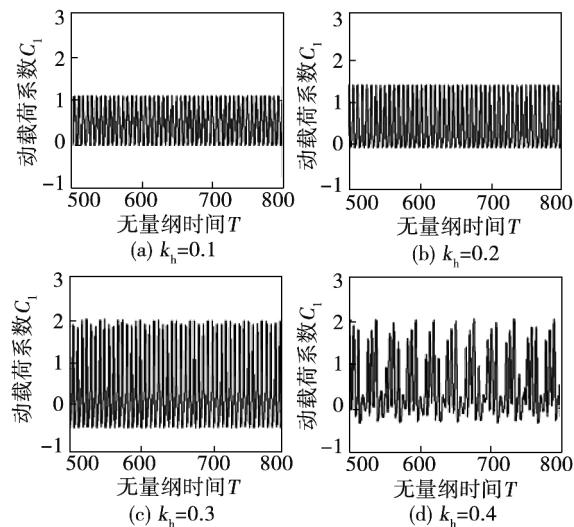


图 3 不同啮合刚度波动值下的齿轮副动载荷系数

Fig. 3 Dynamic load factors of gear pair under different fluctuation values of meshing stiffness

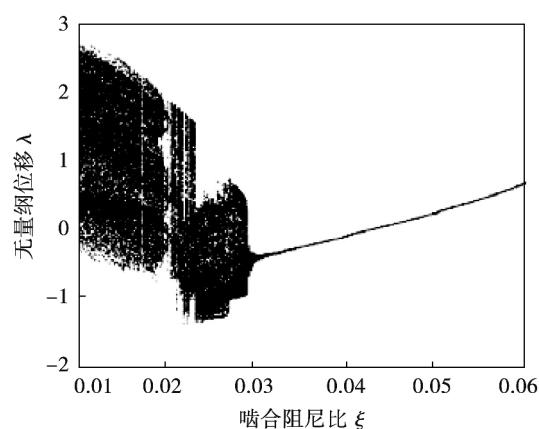


图 4 位移 - 喷合阻尼比分岔图

Fig. 4 Displacement-mesh damping ratio bifurcation diagram

不同喷合阻尼比  $\xi$  下第 1 对齿轮副的动载荷系数如图 5 所示。随  $\xi$  的增大, 动载荷系数逐渐减小, 脱喷现象得到改善, 系统由双边冲击过渡到单边冲击状态, 当  $\xi > 0.03$  时, 动载荷系数不再发生明显变化。

### 2.3 齿侧间隙

取无量纲角频率  $\Omega = 0.5$ , 让无量纲齿侧间隙  $b$  ( $b = b_j/\bar{b}$ ) 在  $0.1 - 2$  之间变化, 得到图 6 所示的系统位移 - 侧隙分岔图。当侧隙  $b$  较小时, 系统处于稳定的单周期响应; 当  $b > 0.69$  时, 系统进入混沌运动, 混沌区域中出现多个周期窗。

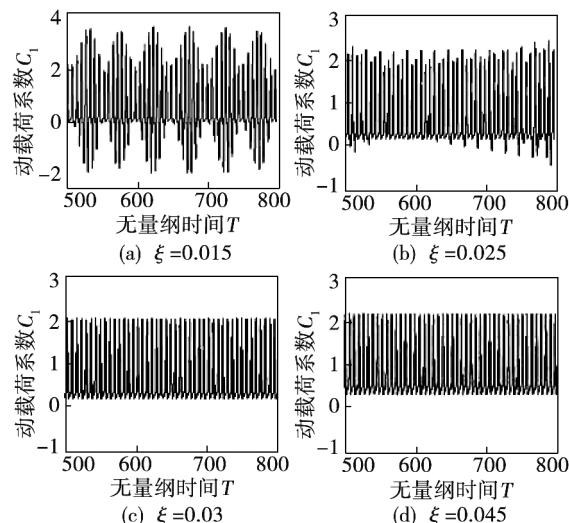


图 5 不同啮合阻尼比下的齿轮副动载荷系数

Fig. 5 Dynamic load factors of gear pair under different mesh damping ratio

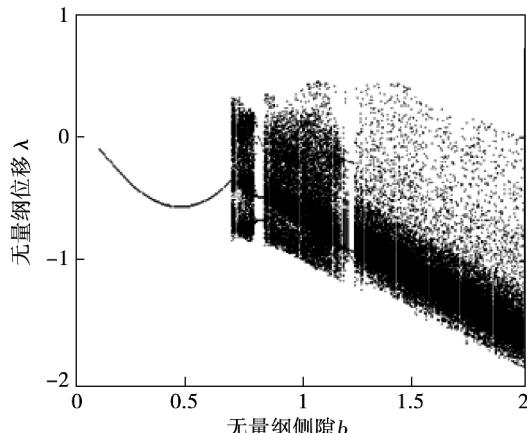


图 6 位移 - 侧隙分岔图

Fig. 6 Displacement-backlash bifurcation diagram

不同侧隙  $b$  下第 1 对齿轮副的动载荷系数如图 7 所示。当  $b$  较小时, 系统处双边冲击状态; 随着  $b$  增大, 动载荷系数随之增大, 出现明显的脱喷现象。当  $b > 1.352$  时, 系统从双边冲击过渡到单边冲击状态, 动载荷系数有所减小。

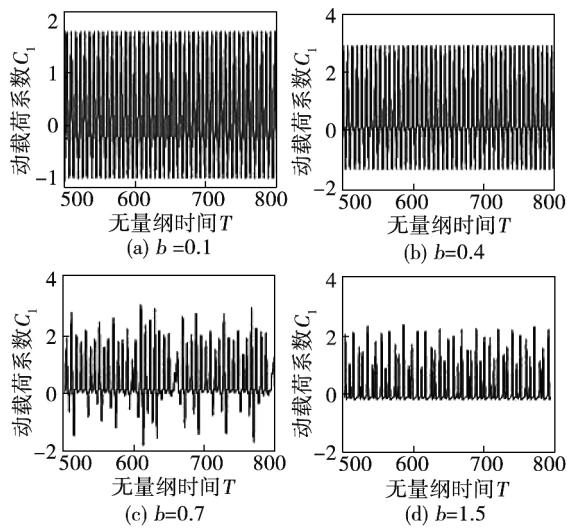


图7 不同侧隙下的齿轮副动载荷系数

Fig. 7 Dynamic load factors of gear pair under different backlashes

#### 2.4 扭转刚度

取无量纲角频率  $\Omega = 0.5$ , 让无量纲扭转刚度  $k_{ij}$  在  $0.1 - 6$  之间变化, 得到图 8 所示的系统位移 – 扭转刚度分岔图。当  $k_{ij}$  较小时, 系统响应较不稳定, 快速经历了单周期、混沌、单周期运动, 并伴随有跳跃现象; 当  $0.9 < k_{ij} < 2.6$  时, 系统处于混沌运动; 随着  $k_{ij}$  的增大, 系统进入了稳定的单周期运动。

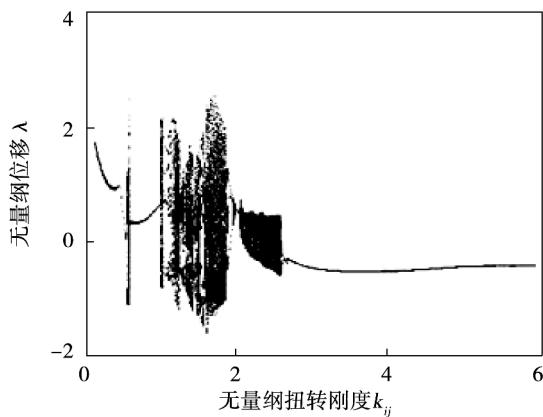


图8 位移 – 扭转刚度分岔图

Fig. 8 Displacement – torsion stiffness bifurcation diagram

不同扭转刚度  $k_{ij}$  下第 1 对齿轮副的动载荷系数如图 9 所示。当  $k_{ij}$  较小时, 齿轮副动载荷系数较大, 出现明显的双边冲击和脱啮现象; 随着  $k_{ij}$  的增大,

齿轮副动载荷系数减小, 齿轮副由双边冲击过渡到单边冲击状态, 脱啮时间减少; 当  $k_{ij} > 3$  时, 动载荷系数不再发生明显变化。

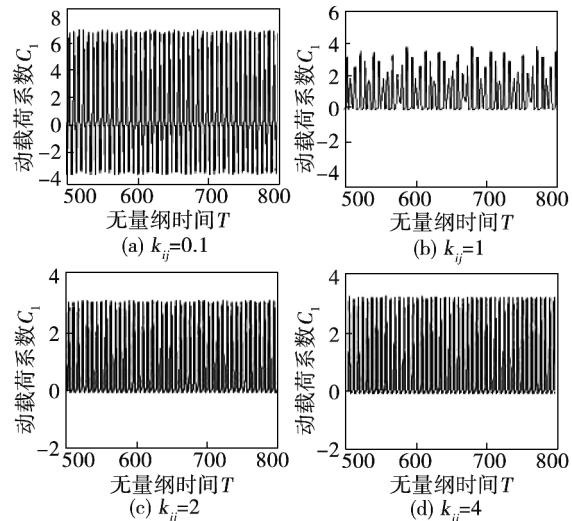


图9 不同扭转刚度下的齿轮副动载荷系数

Fig. 9 Dynamic load factors of gear pair under different torsion stiffness

### 3 结论

(1) 综合考虑轮齿时变啮合刚度、啮合阻尼、传递误差、齿侧间隙以及转速、外载荷等因素, 建立了复杂激励下多级齿轮传动系统扭转振动非线性动力学模型。

(2) 随着齿侧间隙、啮合刚度波动值的增加, 系统响应由稳定的周期运动逐渐过渡到混沌运动状态, 混沌区域常存在多个周期窗, 齿轮副的动载荷系数呈增大趋势, 并出现脱啮现象。

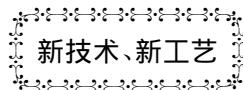
(3) 随着啮合阻尼比、扭转刚度的增加, 系统响应由混沌运动过渡到单周期运动, 齿轮副由双边冲击过渡到单边冲击或无冲击状态, 动载荷系数逐渐减小, 啮合性能得到改善。

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(丛敏 编辑)



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## 具有 BBEP – 1000 的核电站高温过滤工艺发展的问题和展望

据《Теплоэнергетика》2013 年 7 月刊报道 加里宁核技术动力研究所和加里宁核电站的专家分析了具有 BBEP – 1000 压水堆( 我国连云港核电站装用了 4 台 ,每台电功率为 1 000 MW 的购自俄罗斯的 BBEP – 1000 压水堆核电机组的核电站) 的核电站高温过滤工艺发展的问题和前景。

列出了在 BBEP – 1000 反应堆 – 回路载热介质高温净化系统过滤材料使用性能恢复方面的工作结果。

完成了在加里宁核电站 3 号发电机组运行的最初几年利用原配设计的工艺系统进行高温吸附剂疏松清洗效果的数量估计。

提出了优化这个工艺过程的方法。阐述了在整个部门范围内提高高温专用水处理系统的效率和安全运行问题的成套解决方案。

所进行的研究结果表明 ,应用提出的工艺过程能达到减少施加在核电站运行和修理人员辐射剂量的目的。

(吉桂明 摘译)

基于 DBEL 技术超临界“W”火焰锅炉狭缝式燃烧器煤粉着火燃烧特性研究 = Study of the Ignition and Combustion Characteristics of Pulverized Coal in the Slit Type Burners of a Supercritical “W”-shaped Boiler Based on the DBEL Technology [刊 汉] GAO Jia-jia , LIU Peng-yuan , XU Peng-zhi , TANG Wen ( Department of Boiler and Combustion Technology , Huadian Electric Power Science Academy , Hangzhou , China , Post Code: 310000) //Journal of Engineering for Thermal Energy & Power. -2015 , 30(5) . -768 -774

In the light of the problems existing in the first domestically-made 600 MW class supercritical W-shaped flame boiler based on the DBEL technology during its operation ,experimentally studied were the ignition and combustion characteristics of pulverized coal in the novel slit type bias separation burner. The test results show that the ignition of the sparse phase in the burner is relatively poor and the combustion stabilization characteristics of the dense phase are susceptible to the air distribution of the burner. Both constitute the main causes for a low operation efficiency and the poor safety of the lower furnace. The test results can offer a certain theoretical basis and guide for operation and retrofitting of supercritical W-shaped flame boilers in operation in China ,including new boiler design and development. **Key words:** DBEL technology ,supercritical , “W”-shaped boiler ,anthracite ,ignition and combustion characteristics

复杂激励下多级齿轮传动系统分岔特性及动载荷分析 = Bifurcation Characteristics and Dynamic Load Analysis of a Multi-stage Gear Transmission System Under a Complex Excitation [刊 汉] LIU Bo , LIN Teng-jiao , WANG Dan-hua ( National Key Laboratory on Mechanical Transmission , Chongqing University , Chongqing , China , Post Code: 400044) , LU He-sheng ( Chongqing Gear Box Co. Ltd. , Chongqing , China , Post Code: 402263) //Journal of Engineering for Thermal Energy & Power. -2015 , 30(5) . -775 -780

In order to study the bifurcation characteristics and dynamic load coefficient of a multi-stage gear transmission system under a complex excitation ,established was a non-linear dynamic model with six degrees of freedom ,including the time-change engagement rigidity ,engagement damping ,transmission error and backlash etc. factors. The 4-5 order step change Runge-Kutta method was used to seek solutions to the non-dimensional dynamic differential equation of the system. The calculation results show that with an increase of the backlash and engagement rigidity ,the system will tend to be unstable and approximately undergo a state changing from a single period and multi-period to chaotic movement and the dynamic load coefficient will assume an ascending tendency and emerge a disengagement phenomenon. When the engagement damping ratio and twist rigidity increase ,the system will gradually tend to be

stable ,transmitting from the chaotic movement to a single period movement ,the dynamic load coefficient gradually decreasing and the gear pair transmitting to a lateral impingement or no-impingement state ,the engagement phenomenon getting improved. **Key words:** gear transmission ,non-linear kinetics ,bifurcation ,chaos ,dynamic load coefficient

**非等压力角节点外啮合齿轮油膜厚度的研究 = Study of the Oil Film Thickness on a Non-equal Pressure Angle Gear Engaged at a Place Beyond the Pitch Point** [刊 汉] LI Xiu-lian ,LIU Wei ,ZHU Fu-xian ( College of Mechanical Engineering ,Jiangsu University of Science and Technology ,Changzhou ,China ,Post Code: 23001) ,ZHANG Jun ( College of Mechanical Engineering ,Anhui University of Technology ,Ma'anshan ,China ,Post Code: 243002) //Journal of Engineering for Thermal Energy & Power. -2015 ,30(5). -781 -786

With a pair of involute straight tooth cylindrical non-equal pressure angle gear engaged at a place beyond the pitch point serving as the object of study ,on the basis of the structural characteristics of non-equal pressure angle gears engaged at a place beyond the pitch point being considered in a comprehensive way ,through an analysis of the engagement process of the gears ,a formulae for calculating the minimum oil film thickness on the surface of the gears was derived and relevant factors influencing the oil film thickness were analyzed. It has been found that compared with conventional gears ,non-equal pressure angle gears enjoy better lubrication performance. To take such measures as to increase the pressure angle ,modification coefficient ,gear ratio and modulus can increase the minimum oil film thickness by 6.09% ,5.46% ,9.63% and 66.63% respectively. **Key words:** gear transmission ,non-equal pressure angle ,engagement at a place beyond the pitch point ,oil film thickness ,simulation analysis

**基于实测边界条件的小型增压锅炉锅筒应力三维有限元分析 = Three-dimensional Finite Element Analysis of the Stress of the Drum Shell of a Small-sized Supercharged Boiler Based on the Boundary Conditions Actually Measured** [刊 汉] SHAO Ya-xi ,XU Wei-yi ,LI Yan-jun ( College of Power and Energy Source Engineering ,Harbin Engineering University ,Harbin ,China ,Post Code: 150001) ,WANG Kun-feng ( CSIC No. 703 Research Institute ,Harbin ,Harbin ,China ,Post Code: 150078) //Journal of Engineering for Thermal Energy & Power. -2015 ,30(5). -787 -791

Established was a three-dimensional solid model for small-sized supercharged boilers and determined were the me-